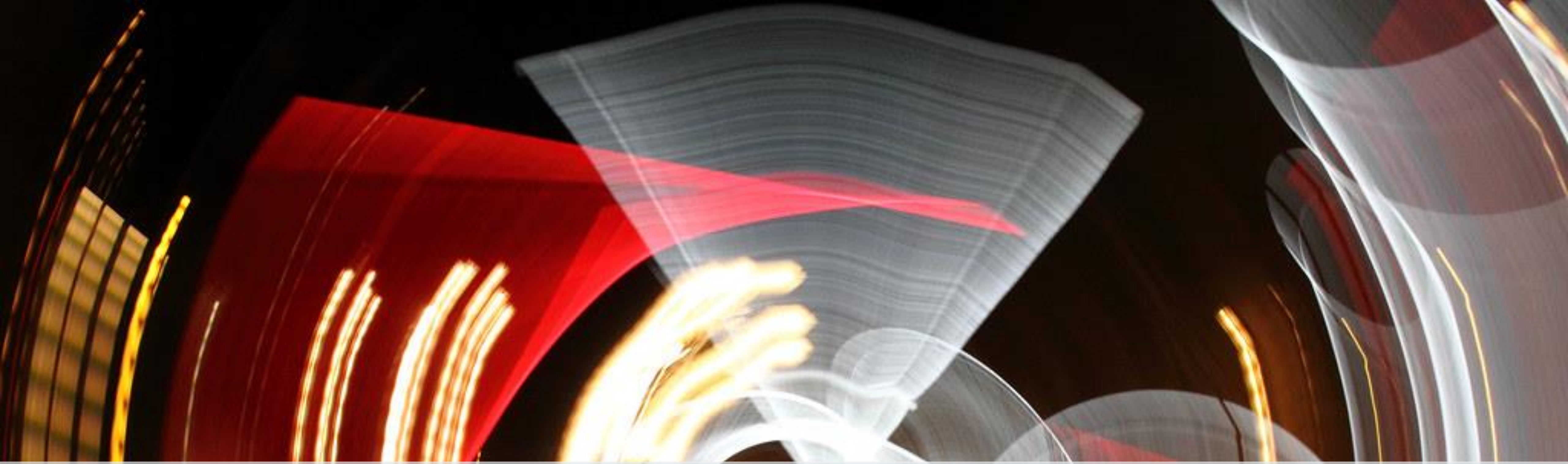

Medan Elektromagnetik

W10

Magnetostatik

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Outline Presentasi

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Magnetostatik

Magnetostatics parallels the preceding one on electrostatics. Stationary charges produce static electric fields, and steady (i.e., non–time-varying) currents produce static magnetic fields. When ∂/∂t = 0, the magnetic fields in a medium with magnetic permeability μ are governed by the second pair of Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0 \quad (10.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (10.2)$$

$$\mathbf{B} = \mu \mathbf{H} \quad (10.3)$$

Furthermore, $\mu = \mu_0$ for most dielectrics and metals (excluding ferromagnetic materials).

J = current density /kerapatan arus.
B = magnetic flux density / kerapatan fluks magnet
H = magnetic field intensity / intensitas medan magnet

Gaya dan Torsi Magnetik

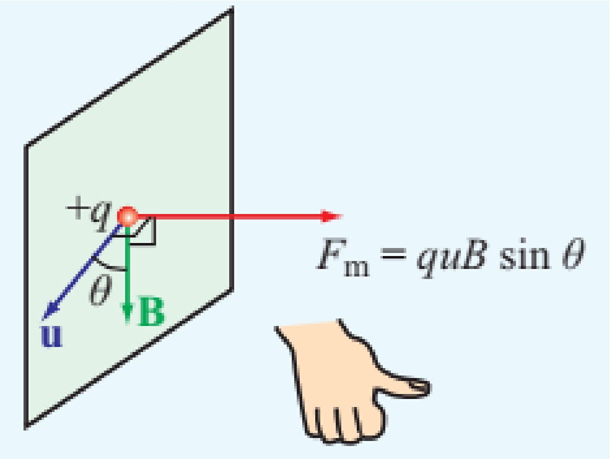
- *We now define the magnetic flux density* **B** *at a point in space in terms of the magnetic force* $\mathbf{F}_m$ *that acts on a charged test particle moving with velocity u through that point.*
- *Experiments revealed that a particle of charge* **q** *moving with velocity* **u** *in a magnetic field experiences a magnetic force* $\mathbf{F}_m$ *given by:*

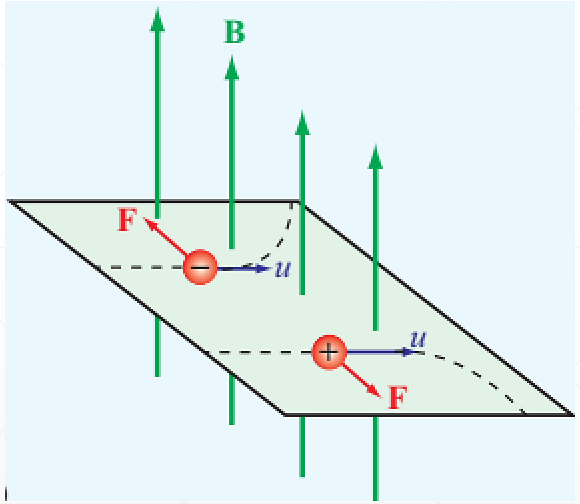
$$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B} \qquad (10.4)$$

$$F_m = quB \sin\theta \qquad (10.5)$$

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = q\mathbf{E} + q\mathbf{u} \times \mathbf{B} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \qquad (10.6)$$

θ = sudut yang terbentuk antara **u** dan **B**





Gambar 10.1 (atas) *The direction of the magnetic force exerted on a charged particle moving in a magnetic field is (a) perpendicular to both* **B** *and* **u** *and* (bawah) *depends on the charge polarity (positive or negative).*

Attribute	Electrostatics	Magnetostatics
Sources	Stationary charges ρ_v	Steady currents $\mathbf{J}$
Fields and Fluxes	$\mathbf{E}$ and $\mathbf{D}$	$\mathbf{H}$ and $\mathbf{B}$
Constitutive parameter(s)	ε and σ	μ
Governing equations		
• **Differential form**	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = 0$	$\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$
• **Integral form**	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$ $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ $\oint_C \mathbf{H} \cdot d\mathbf{l} = I$
Potential	Scalar V, with $\mathbf{E} = -\nabla V$	Vector $\mathbf{A}$, with $\mathbf{B} = \nabla \times \mathbf{A}$
Energy density	$w_e = \frac{1}{2}\varepsilon E^2$	$w_m = \frac{1}{2}\mu H^2$
Force on charge q	$\mathbf{F}_e = q\mathbf{E}$	$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$
Circuit element(s)	C and R	L

Hukum Biot-Savart

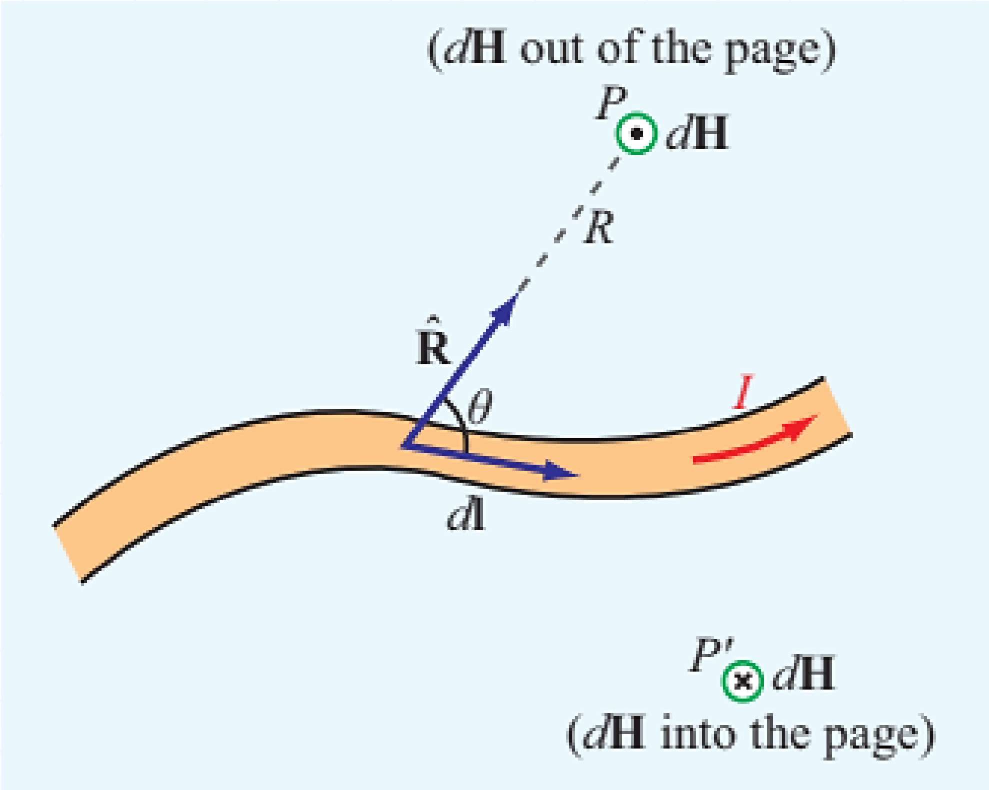
*The Biot–Savart law states that the differential magnetic field d**H** generated by a steady current I flowing through a differential length vector d**l** is:*

$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad \text{(A/m)} \qquad (10.7)$$

To determine the total magnetic field H due to a conductor of finite size, need to sum up the contributions due to all the current elements making up the conductor. Hence, the Biot– Savart law becomes

$$\mathbf{H} = \frac{I}{4\pi}\int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad \text{(A/m)} \qquad (10.8)$$

l = is the line path along which I exists.



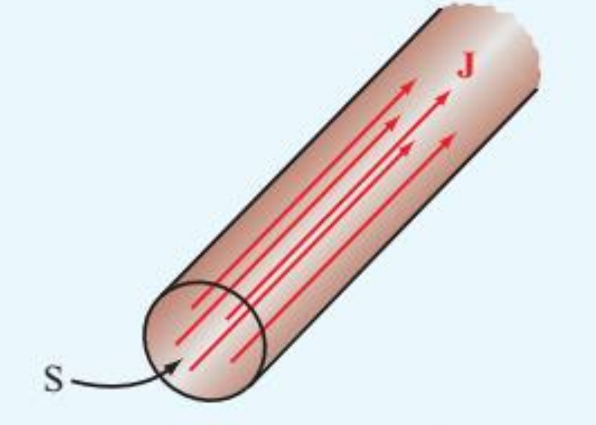
Gambar 10.2 Medan magnet $d\mathbf{H}$ yang dihasilkan dari elemen arus I, $d\mathbf{l}$. Arah medan terinduksi pada titik P berlawanan dengan titik terinduksi P’.

Magnetic Field Due to Surface and Volume Current Distributions

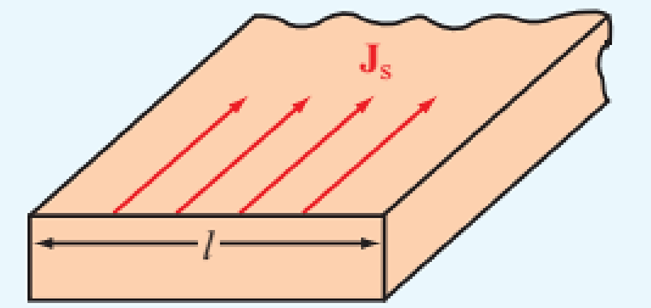
Hukum Biot–Savart dapat diekspresikan dalam bentuk *distributed current sources (gbr 10.3) such as the volume current density J, measured in (A/m²)*, atau *surface current density Js, measured in (A/m). The surface current density Js* memanfaatkan arus yang mengalir pada permukaan konduktor dalam bentuk *sheet of effectively zero thickness*.

When current sources are specified in terms of J_s over a surface S or in terms of J over a volume υ, we can use the equivalence given by

$$I\, d\mathbf{l} \leftrightarrow \mathbf{J}_s ds \leftrightarrow \mathbf{J} dv \qquad (10.9)$$



(a) Volume current density **J** in A/m²



(b) Surface current density $\mathbf{J}_s$ in A/m

Gambar 10.3 (a) Total arus yang melewati penampang melintang (*cross section*) S dari silinder adalah $I = \int_S \mathbf{J} \cdot d\mathbf{s}$. (b) Total arus yang mengalir pada permukaan konduktor adalah $I = \int_l \mathbf{J}_s \cdot d\mathbf{l}$.

$$\mathbf{H} = \frac{1}{4\pi} \int_S \frac{\mathbf{J}_s \times \hat{\mathbf{R}}}{R^2}\, ds,$$

(surface current)

$$\mathbf{H} = \frac{1}{4\pi} \int_\upsilon \frac{\mathbf{J} \times \hat{\mathbf{R}}}{R^2}\, d\upsilon.$$

(volume current)

$$R = r\csc\theta,$$

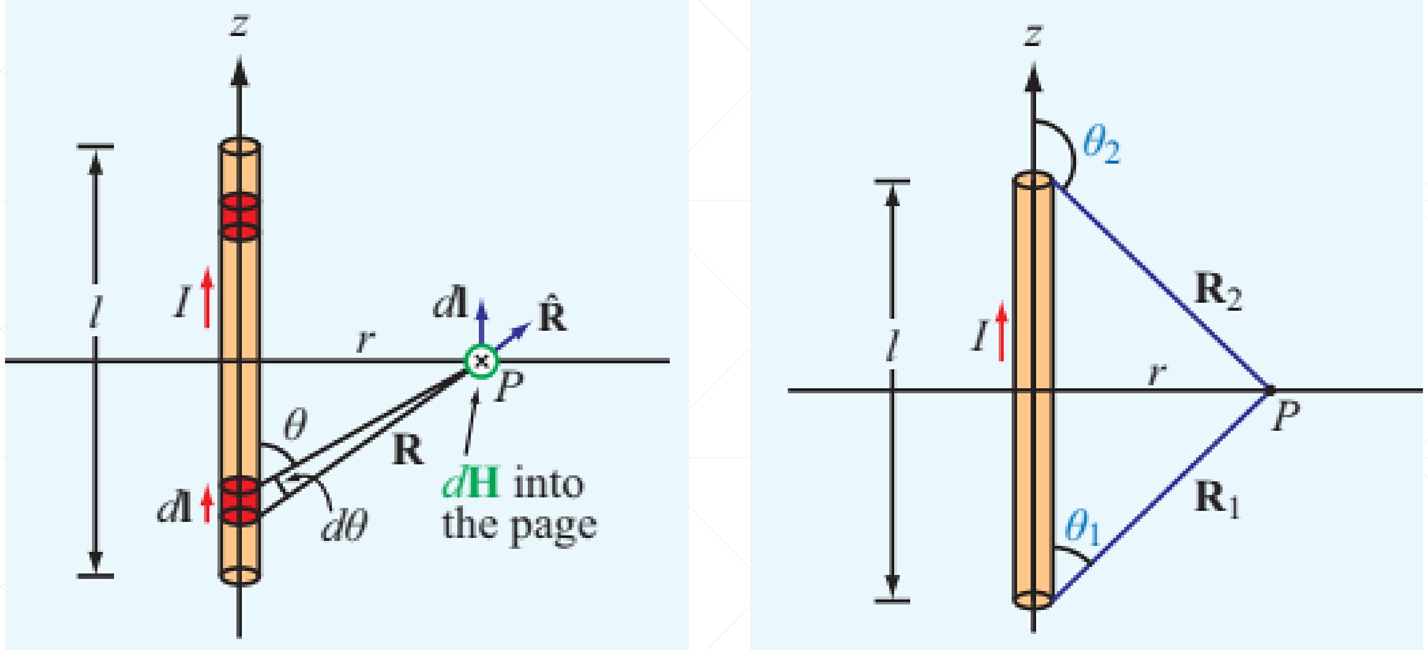
$$z = -r\cot\theta,$$

$$dz = r\csc^2\theta \, d\theta.$$

Dari gambar 10.4 (kanan)

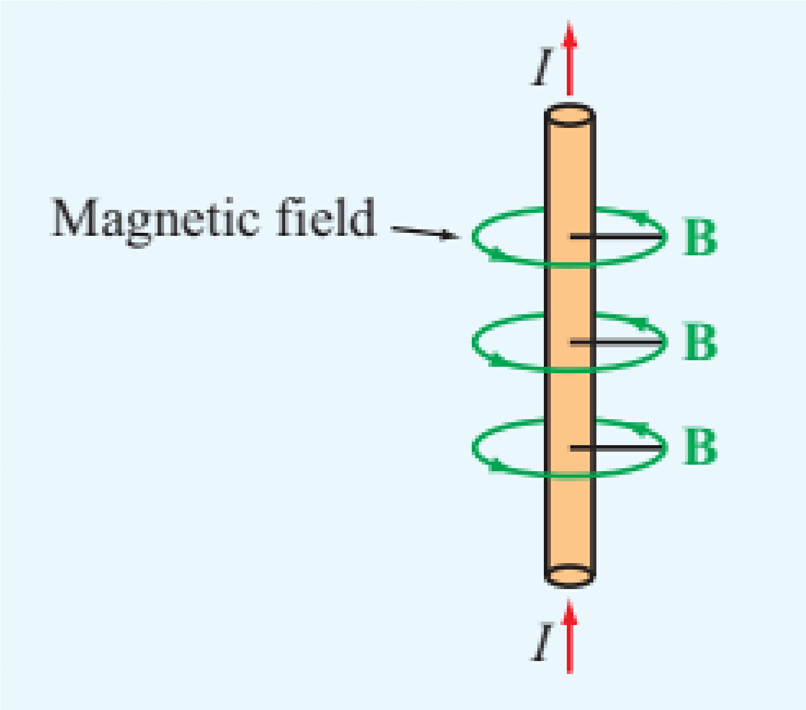
$$\cos\theta_1 = \frac{l/2}{\sqrt{r^2 + (l/2)^2}},$$

$$\cos\theta_2 = -\cos\theta_1 = \frac{-l/2}{\sqrt{r^2 + (l/2)^2}}.$$



Gambar 10.4 *Linear conductor of length l carrying a current I.*
(kiri) *The field d**H** at point P due to incremental current element d**l**.*
(kanan) *Limiting angles θ_1 and θ_2, each measured between vector I d**l** and the vector connecting the end of the conductor associated with that angle to point P.*

◭ *This is a very important and useful expression to keep in mind. It states that in the neighborhood of a linear conductor carrying a current I, the induced magnetic field forms concentric circles around the wire (gbr 10.5), and its intensity is directly proportional to I and inversely proportional to the distance r.* ◭



Gambar 10.5 Medan magnet berada di konduktor panjang yang membawa arus linear.

$$\mathbf{B} = \mu_0 \mathbf{H}$$

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 \boldsymbol{I} l}{2\pi r\sqrt{4r^2 + l^2}} \qquad (10.10)$$

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 \boldsymbol{I}}{2\pi r} \qquad (10.11)$$

Formulasi 10.11 digunakan untuk infinitely long wire (kondisi $l >> r$)

Magnetic Field of a Magnetic Dipole

$$\mathbf{H} = \hat{\mathbf{z}} \frac{Ia^2}{2\pi|z|^3} \quad \text{untuk } |z| >> a \qquad (10.12)$$

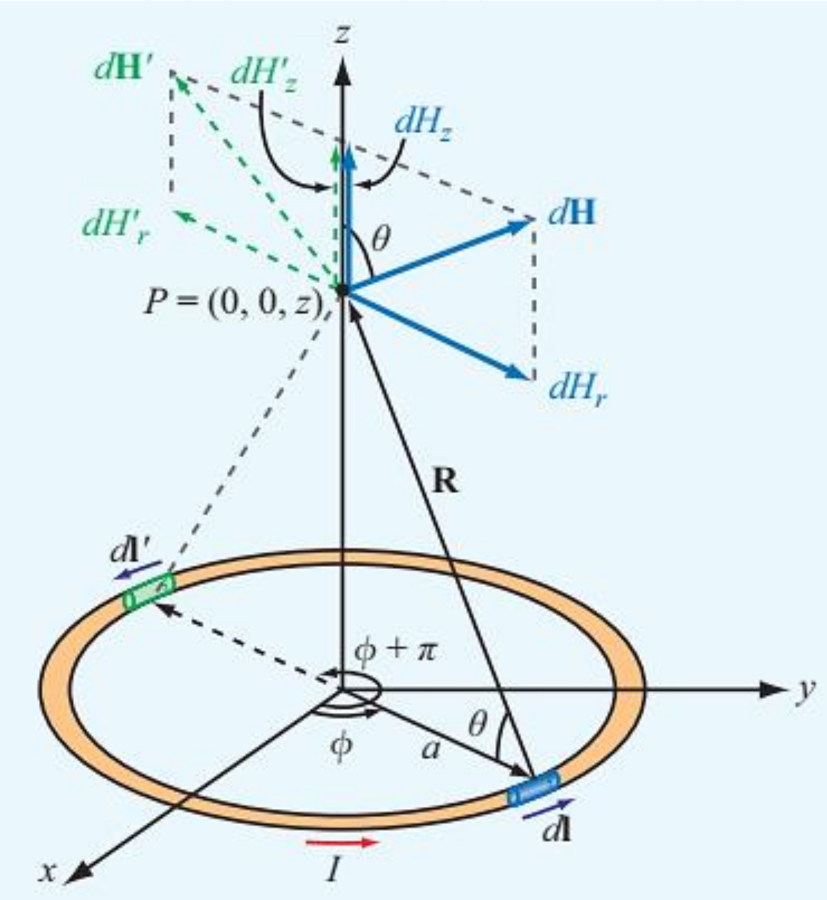
In view of the definition given by :

$\mathbf{m} = \hat{\mathbf{n}}NIA = \hat{\mathbf{n}}m$ (A.m²)

for the magnetic moment m of a current loop, a single-turn loop situated in the x–y plane (Gbr. 10.6) *has* magnetic moment $\mathbf{m} = \hat{\mathbf{z}}m$ with $\mathbf{m} = I\pi a^2$. *Consequently, Eq. (10.12) may be expressed as:*

$$\mathbf{H} = \hat{\mathbf{z}} \frac{m}{2\pi|z|^3} \quad \text{untuk } |z| >> a \qquad (10.13)$$

$$\mathbf{H} = \frac{m}{4\pi R^3}\left(\hat{\mathbf{R}} 2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \quad \text{untuk } R >> a \qquad (10.14)$$

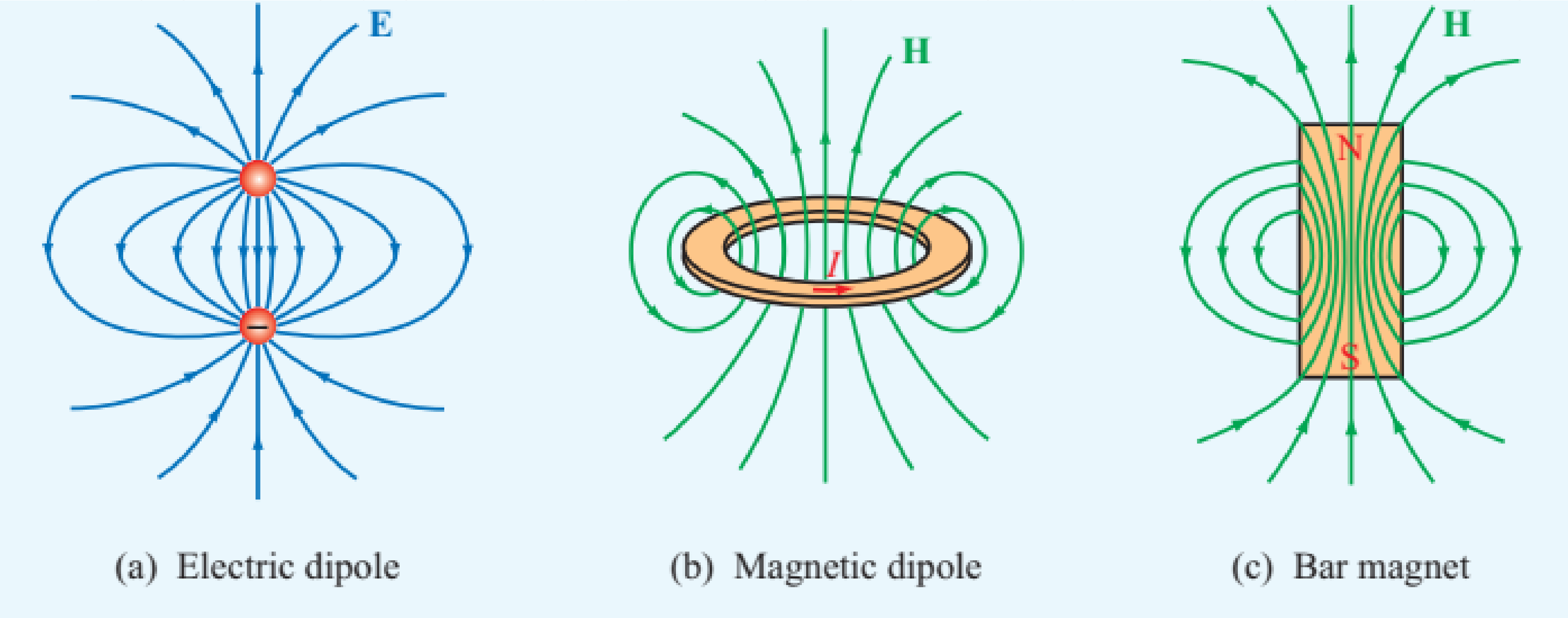


Gambar 10.6 Circular loop carrying a current I

*From eq 10.13 applies to a point P far away from the loop and on its axis. Had we solved for **H** at any distant point P = (R,θ,φ) in a spherical coordinate system with R the distance between the center of the loop and point P, we would have obtained the expression 10.14*

NIA= momen magnetik

◭ A current loop with dimensions much smaller than the distance between the loop and the observation point is called a magnetic dipole.
This is because the pattern of its magnetic field lines is similar to that of a permanent magnet as well as to the pattern of the electric field line of the electric dipole (Gbr. 10.7). ◭



Gambar 10.7 Patterns of (a) the electric field of an electric dipole, (b) the magnetic field of a magnetic dipole, and (c) the magnetic field of a bar magnet.
Far away from the sources, the field patterns are similar in all three cases.