Teknik Digital

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Outlines

Simplification Techniques

- Terdapat suatu cara yang bukan merupakan aplikasi dari hukum-hukum dan teorema bilangan Boolean.
- **Tujuan dari proses simplifikasi atau penyederhanaan adalah untuk menghasilkan suatu ekspresi yang memiliki sedikit bentuk istilah bilangan.** Mendapatkan ekspresi dengan jumlah literal yang sedikit merupakan tujuan lainnya.
- *If there is more than one possible solution with the same number of terms, the one having the minimum number of literals is the choice.*
- Teknik-teknik tersebut adalah Quine-McCluskey metode tabel dan metode peta Karnaugh.
- Namun sebelum menjalankannya, harus terlebih dahulu mengetahui ekspresi Boolean dalam bentuk **Sum of Product (SoP)** dan **Product of Sum (PoS)**.

Sum of Product (SOP)

- **Sum of Product** is the abbreviated form of **SOP**. Sum of product form is a form of expression in Boolean algebra in which different product terms of inputs are being summed together. This product is not arithmetical multiply but it is Boolean logical AND and the Sum is Boolean logical OR.
- There are few different forms of Sum of Product:
  - **Canonical** SOP Form
  - **Non-Canonical** SOP Form
  - **Minimal** SOP Form
- To understand better about SOP, we need to know about **min term**.

6.1.1

Minterm

- *Minterm means the term* ***that is true for a minimum number of combination of inputs****. That is true for only one combination of inputs.*
- *Since AND gate also gives True only when all of its inputs are true so we can say minterms are AND of input combinations like in the table given.*
- *3 inputs have 8 different combinations. Each combination has a minterms denoted by small m and its decimal combination number written in subscript.*
- *Each of these minterms will be only true for the specific input combination.*

$$Y = \overline{A}\,.\overline{B}.\overline{C} + \overline{A}.B.C + A.B.\overline{C} + A.\overline{B}.C$$

A	B	C	Y	Min term
0	0	0	1	$m_0 = \overline{A}\,\overline{B}\,\overline{C}$
0	0	1	0	$m_1 = \overline{A}\,\overline{B}\,C$
0	1	0	0	$m_2 = \overline{A}\,B\,\overline{C}$
0	1	1	1	$m_3 = \overline{A}\,B\,C$
1	0	0	0	$m_4 = A\,\overline{B}\,\overline{C}$
1	0	1	1	$m_5 = A\,\overline{B}\,C$
1	1	0	1	$m_6 = A\,B\,\overline{C}$
1	1	1	0	$m_7 = A\,B\,C$

Diambil yang ber-angka 1

Canonical SOP Form

- Merupakan model standar dari Sum of Product.
- *This is also known as Sum of Min terms or Canonical disjunctive normal form (CDNF).*
- *The "canonical" means "standardized" and "disjunctive" means "Logical OR union".*
- *Canonical SOP expression is represented by summation sign ∑ and minterms in the braces for which the output is true.*

For this function the canonical SOP expression is:

$$F = \sum(m_1, m_2, m_3, m_5)$$

Which means that the function is true for the min terms

{1, 2, 3, 5}.

By expanding the summation we get:

$$F = m_1 + m_2 + m_3 + m_5$$

Now putting min terms in the expression:

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$$

Canonical form contains all inputs either complemented or non-complemented in its product terms.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Non-Canonical SOP Form

- As the name suggests, this form is the non-standardized form of SOP expressions.
- The product terms are not the min terms but they are simplified.

Let's take the previous function in canonical form as an example.

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$$

$$F = \bar{A}\bar{B}C + \bar{A}B(\bar{C} + C) + A\bar{B}C$$

$$F = \bar{A}\bar{B}C + \bar{A}B(1) + A\bar{B}C$$

$$F = \bar{A}\bar{B}C + \bar{A}B + A\bar{B}C$$

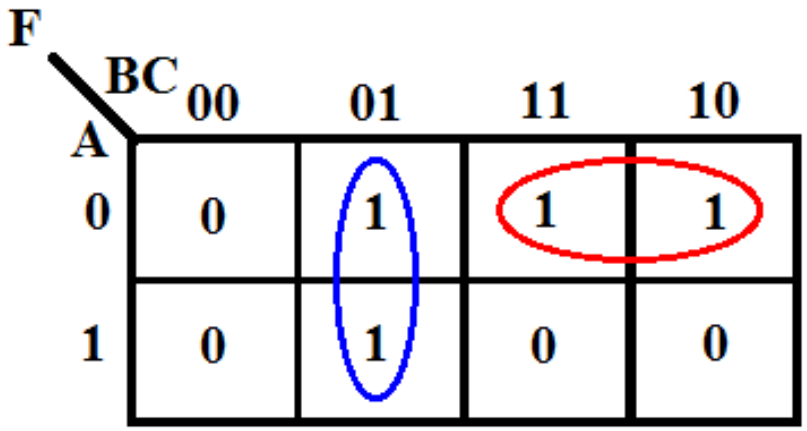
Coba perhatikan beda-nya di bagian mana?

The last expression is still in **Sum of Product form but it obtained by non-canonical or non-standardized form**.

Minimal SOP Form

- *This form is the most simplified SOP expression of a function. It is also a form of non-canonical form.*
- *Minimal SOP form can be made using Boolean algebraic theorems but it is very easily made using Karnaugh map (K-map).*
- *Minimal SOP form is preferred because it uses the minimum number of gates and input lines. Commercially beneficial because of its compact size, fast speed, and low fabrication cost.*
- *Let's take an example of the function given above in canonical form (next slide).*

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



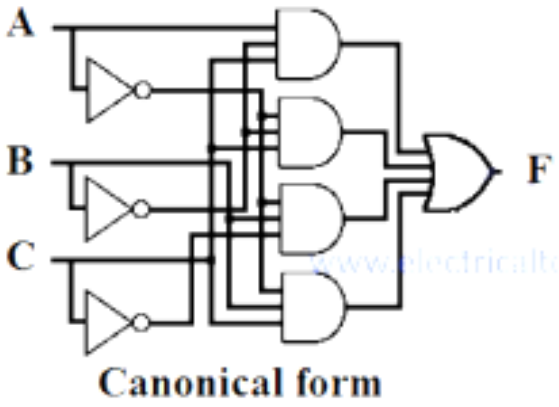
According to the K-map, the output expression will be:

$$F = \bar{B}C + \bar{A}B$$

This expression requires only two 2-input AND gates & one 2-input OR gate. However, the canonical form needs four 3-input AND gates & one 4-input OR gate, which is relatively more costly than minimal form implementation.

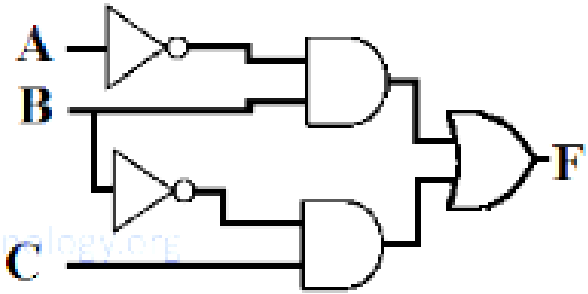
Schematic Design of Sum Of Product (SOP)

- Example of designs of canonical and minimal SOP expression for a function is given below.



Canonical form

$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$



Minimal form

$F = \bar{B}C + \bar{A}B$

Mana yang lebih sederhana?
Apa alasannya?

Product of Sum (POS)

- *The product of Sum form is a form in which products of different sum terms of inputs are taken.*
- *These are not arithmetic product and sum but they are logical Boolean AND and OR respectively.*
- *To better understand about Product of Sum, we need to know about* ***maxterm****.*
- *Three types of POS:*
  - *Canonical POS Form*
  - *Non – Canonical Form*
  - *Minimal POS Form*

Maxterm

- *Maxterm means the term or expression that is true for a maximum number of input combinations or that is false for only one combination of inputs.*
- *Since OR gate also gives false for only one input combination.* ***Maxterm is OR of either complemented or non-complemented inputs.***
- *Max terms for 3 input variables are given side.*

A	B	C	Max term
0	0	0	$M_0 = A + B + C$
0	0	1	$M_1 = A + B + \bar{C}$
0	1	0	$M_2 = A + \bar{B} + C$
0	1	1	$M_3 = A + \bar{B} + \bar{C}$
1	0	0	$M_4 = \bar{A} + B + C$
1	0	1	$M_5 = \bar{A} + B + \bar{C}$
1	1	0	$M_6 = \bar{A} + \bar{B} + C$
1	1	1	$M_7 = \bar{A} + \bar{B} + \bar{C}$

Diambil yang ber-angka 0

- *In maxterm, each input is complemented because Maxterm gives '0' only when the mentioned combination is applied and* ***Maxterm is complement of minterm.***

$M_3 = \bar{m}_3$

$M_3 = (\bar{A}BC)'$

$M_3 = A + \bar{B} + \bar{C}$ DE Morgan's law

- Which is why for A = 0 Maxterm consist A & for A = 1 Maxterm consist .

Canonical POS

- It is also known as Product of Max term or Canonical conjunctive normal form (CCNF). Canonical means standard and conjunctive means intersection.
- In this form, Maxterms are AND together for which output is false.
- Canonical POS expression is represented by ∏ (pi) and Maxterms for which output is false in brackets as shown in the example given table (next slide).

$$F = \prod (M_0, M_4, M_6, M_7)$$

Expanding the product

$$F = M_0.M_4.M_6.M_7$$

Putting Max terms:

$$F = (A+B+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Non-Canonical Form

- The product of sum expression that is not in standard form is called non-canonical form.
- Let’s take the previous-given function as an example:

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$$

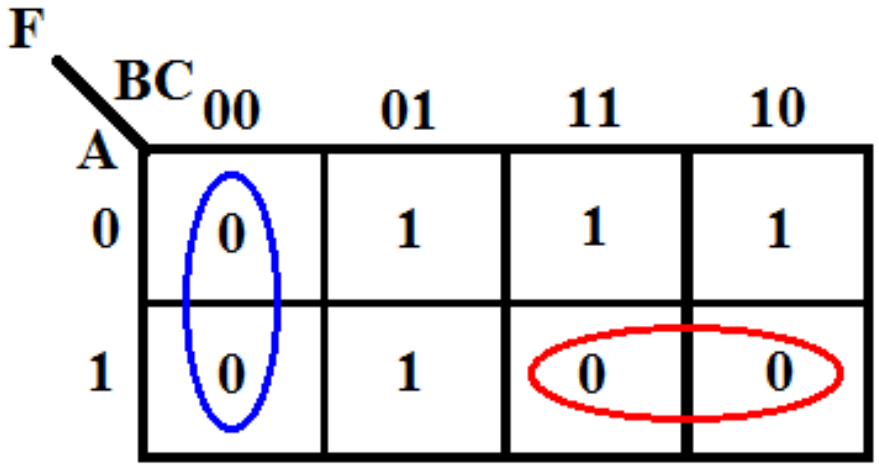
Same but inverted terms eliminates from two Max terms and form a single term to prove it here is an example:

Minimal POS Form

- *This is the most simplified and optimized form of a POS expression which is non-canonical.*
- *Minimal Product of Sum form can be achieved using Boolean algebraic theorems like in the non-canonical example given before.*
- *Another method of achieving minimal POS form is by using Karnaugh map which is comparatively easier than using Boolean algebraic theorems.*
- *Minimal POS form uses less number of inputs and logic gates during its implementation, that's why they are being preferred over canonical form for their compact, fast and low-cost implementation.*

K-map of the function

Diambil yang ber-angka 0

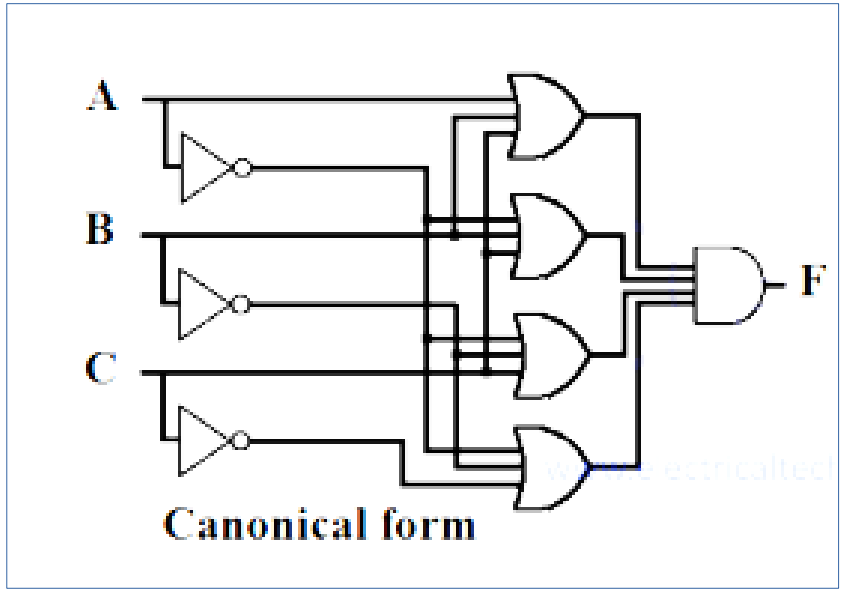


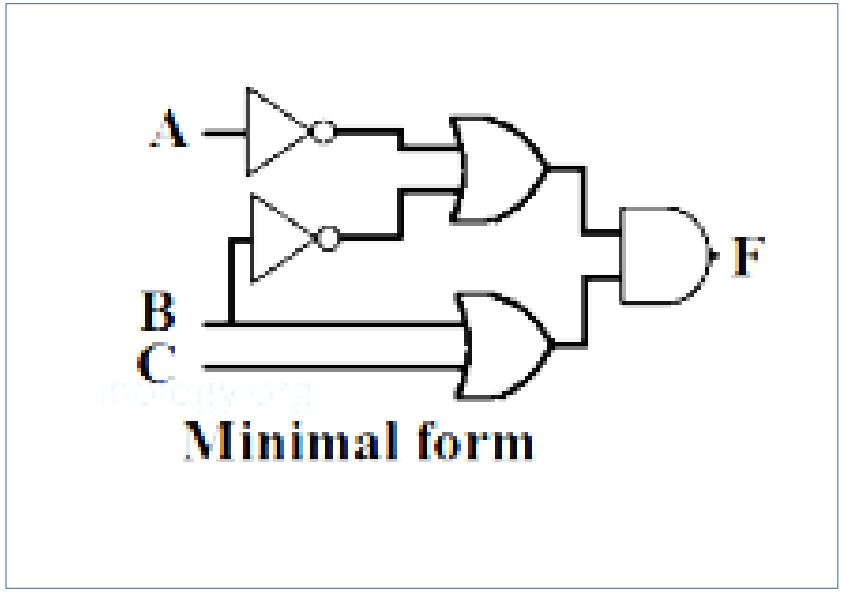
Minimal expression using K-map

$$F = (B+C)(\bar{A}+\bar{B})$$

- *The achieved expression is the minimal product of sum form.*
- *It is still Product of Sum expression But it needs only 2 inputs two OR gates and a single 2 input AND gate.*
- *However, the canonical form needs 4 OR gates of 3 inputs and 1 AND gate of 4 inputs.*

Schematic Design of Product of Sum (POS)





$$F = (A+B+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$$

$$F = (B+C)\,(\bar{A}+\bar{B})$$

Mana yang lebih sederhana?
Apa alasannya?

Zoom Meeting

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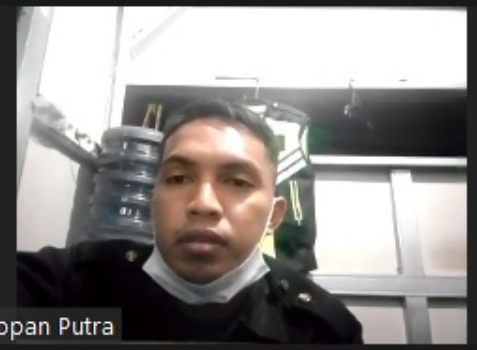

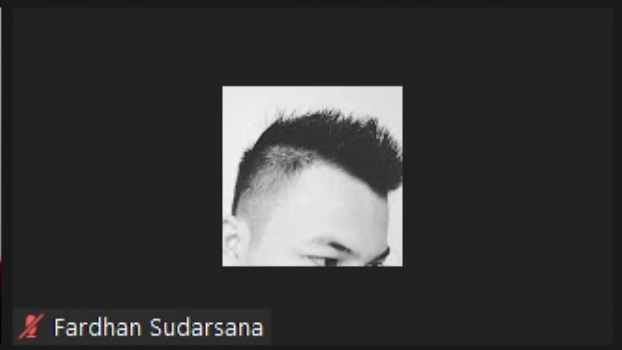

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