Rangkaian Listrik II

Pertemuan 4

Bilangan Kompleks

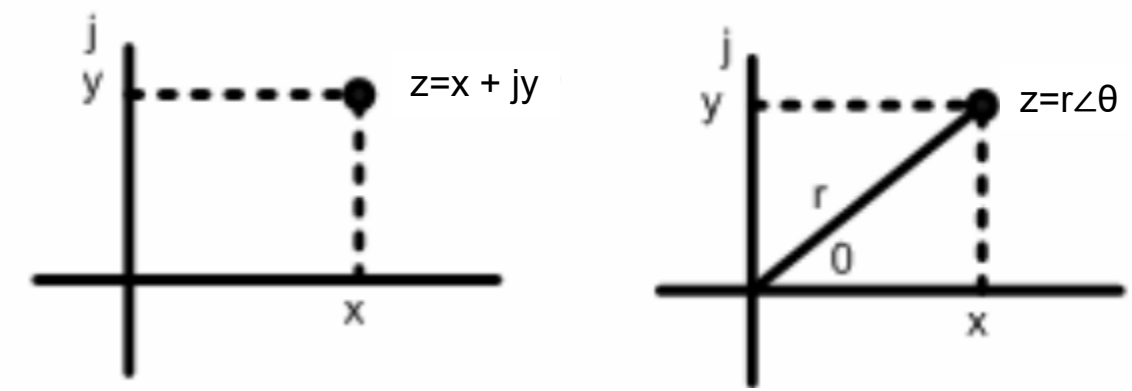
Bilangan yang terdiri dari harga real (nyata) dan harga imajiner (khayal)
Contoh :

$$\boldsymbol{z = x + jy}$$

dimana :

$j = \sqrt{-1}\ atau\ j^2 = -1$

Grafik bilangan kompleks :



Bentuk - bentuk bilangan kompleks :

1. Bentuk Kartesian / Rectanguler

$$z = x + jy \quad \rightarrow \quad r = \sqrt{x^2 + y^2}, \quad \theta = tan^{-1}\frac{y}{x}$$

2. Bentuk Polar

$$z = r\angle\theta$$

dimana : $x = r\cos\theta$

$y = r\,sin\,\theta$

3. Bentuk Eksponensial

$$z = re^{j\theta}$$

dimana : $x + jy = r\,cos\,\theta + jr\sin\theta = r(\cos\theta + j\sin\theta) = re^{j\theta}$

4. Bentuk Trigonometri

$$z = r(\cos\theta + j\sin\theta)$$

Konjugate bilangan kompleks

$$z \rightarrow z^*$$

$$z = x + jy \rightarrow z^* = x - jy$$

$$z = r\angle\theta \rightarrow z^* = r\angle -\theta$$

$$z = re^{j\theta} \rightarrow z^* = re^{-j\theta}$$

$$z = r(\cos\theta + j\sin\theta) \rightarrow z^* = r(\cos\theta - j\sin\theta)$$

Jumlah dan selisih bilangan kompleks

$$z_1 = x_1 + jy_1$$

$$z_2 = x_2 + jy_2$$

$$Z_1 + Z_2 = (X_1 + jY_1) + (X_2 + jY_2) = (X_1 + X_2) + j(Y_1 + Y_2)$$

$$Z_1 - Z_2 = (X_1 + jY_1) - (X_2 + jY_2) = (X_1 - X_2) + j(Y_1 - Y_2)$$

Arus dan Tegangan Sinusoidal

Arus sinusoidal :

Tegangan yang melewati elemen pasif jika arusnya sinusoidal

elemen	**i**	$i = I_m \sin \omega t$	$i = I_m \cos \omega t$
R	$V_R = R.i$	$V_R = R.I_m \sin \omega t$	$V_R = R.I_m \cos \omega t$
L	$V_L = L.\frac{di}{dt}$	$V_L = \omega.L.I_m \cos \omega t$	$V_L = \omega.L.I_m(-\sin \omega t)$
C	$V_C = \frac{1}{C}\int i dt$	$V_C = \frac{I_m}{\omega C}(-\cos \omega t)$	$V_C = \frac{I_m}{\omega C}\sin \omega t$

Tegangan sinusoidal :

Arus pada elemen pasif jika tegangannya sinusoidal

elemen	**v**	$V = V_m \sin \omega t$	$V = V_m \cos \omega t$
R	$i_R = \frac{V}{R}$	$i_R = \frac{V_m}{R} \sin \omega t$	$i_R = \frac{V_m}{R} \cos \omega t$
L	$i_L = \frac{1}{L} \int v dt$	$i_L = \frac{V_m}{\omega L} (-\cos \omega t)$	$i_L = \frac{V_m}{\omega L} \sin \omega t$
C	$i_C = C \frac{dV}{dt}$	$i_C = \omega C V_m \cos \omega t$	$i_C = \omega C V_m (-\sin \omega t)$

Sudut Phasa

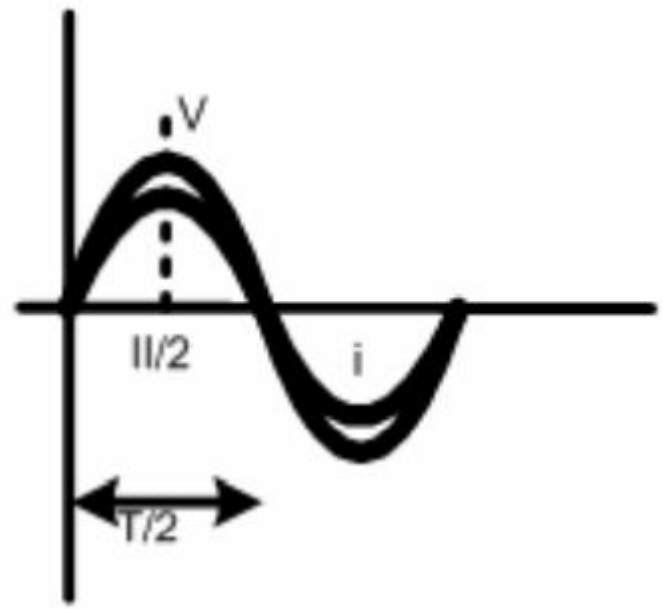
Pengaruh gelombang AC pada elemen R :

$$i = I_m \sin \omega t \Rightarrow I = I_m \angle 0^o$$

$$V_R = RI_m \sin \omega t \Rightarrow V_R = RI_m \angle 0^o$$

phasanya..sama

$$Magnitude\, impedansi.. |Z| = R$$



Pengaruh gelombang AC pada elemen L :

$$i = I_m \sin \omega t \Rightarrow I = I_m \angle 0^o$$

$$V_L = \omega L I_m \cos \omega t = \omega L I_m \sin\left(\omega t + 90^o\right)$$

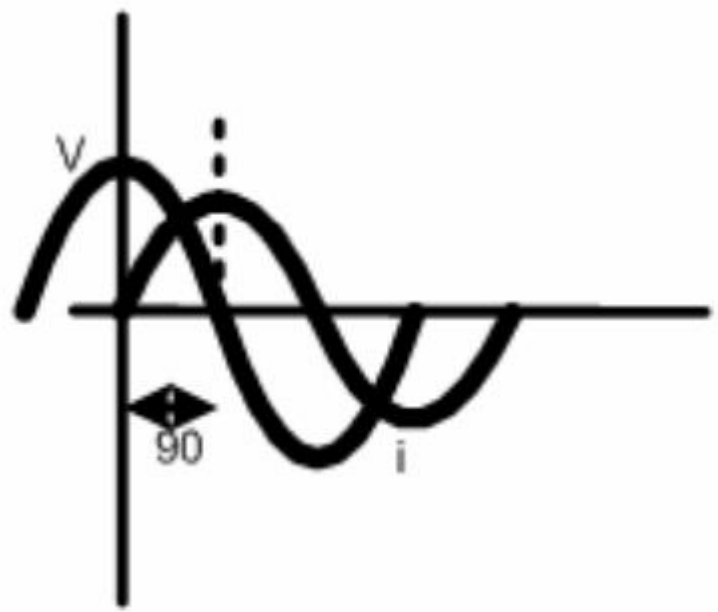
$$\Rightarrow V_L = \omega L I_m \angle 90^o$$

Arus tertinggal dibanding tegangan sebesar 90^o

$\rightarrow$ *arus lagging*

$$Z = \frac{V_L}{I} = \frac{\omega L I_m \angle 90^\circ}{I_m \angle 0^\circ}$$

$$Z = \omega L \angle 90^\circ = j\omega L$$



Pengaruh gelombang AC pada elemen C :

$$i = I_m \sin \omega t \Rightarrow I = I_m \angle 0^o$$

$$V_C = \frac{I_m}{\omega C}\left(-\cos \omega t\right) = \frac{I_m}{\omega C}\sin\left(\omega t - 90^o\right)$$

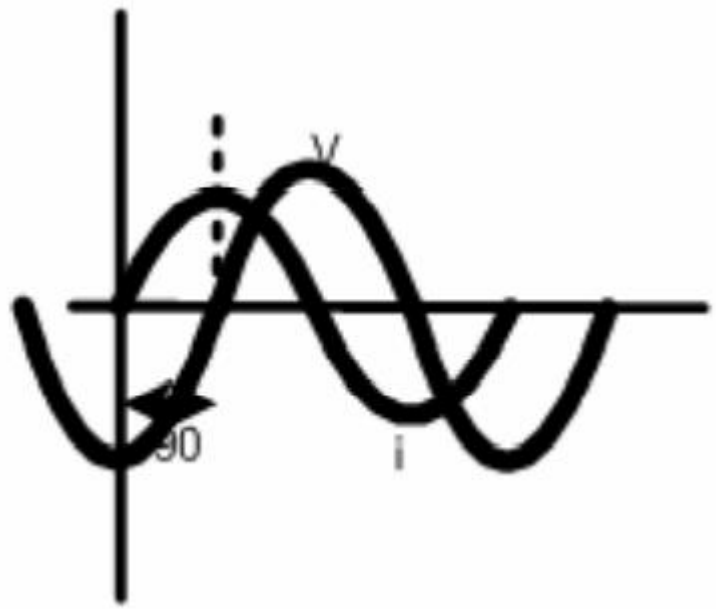
$$\Rightarrow V_C = \frac{I_m}{\omega C} \angle -90^\circ$$

Arus mendahului dibanding tegangan sebesar 90^o

$\rightarrow$ *arus leading*

$$Z = \frac{V_C}{I} = \frac{\frac{I_m}{\omega C} \angle -90^\circ}{I_m \omega \angle 0^\circ}$$

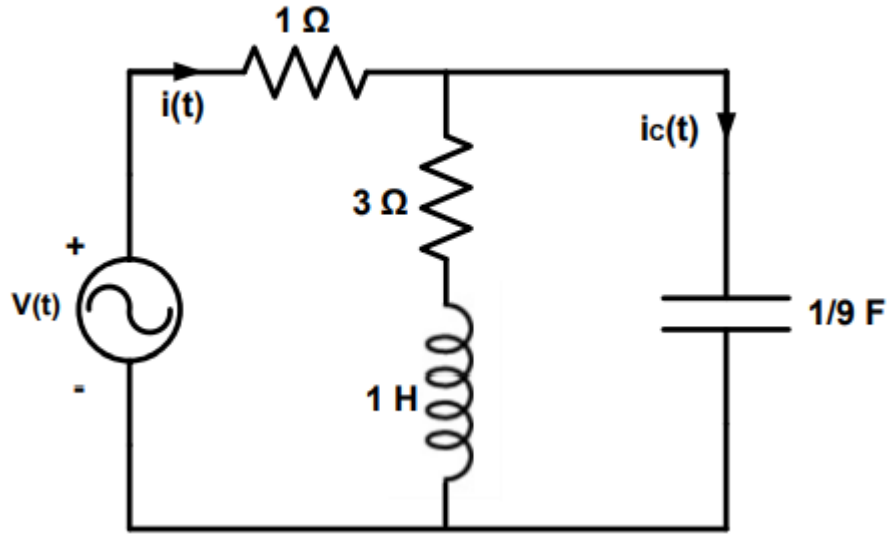
$$Z = \frac{1}{\omega C} \angle -90^\circ = -\frac{j}{\omega C} = \frac{1}{j\omega C}$$



Contoh Soal

1. Dalam rangkaian elektrik, diketahui sumber tegangan :

$$v(t) = 5\sin(3t) \text{ volt}$$



a). Hitung arus efektif Ief dan arus sesaat i(t) pada rangkaian
b). Hitung arus efektif Ief dan arus sesaat i(t) yang melewati kapasitor