Resonansi

Suatu rangkaian dikatakan beresonansi ketika tegangan terpasang V dan arus yang dihasilkan I dalam kondisi satu phasa.
Misalkan :

$V = A\angle\alpha°$

$I = B\angle\beta°$

Dalam kondisi satu phasa : $\alpha° = \beta°$, sehingga :

$$Z = \frac{V}{I} = \frac{A\angle\alpha°}{B\angle\beta°} = \frac{A}{B}\angle(\alpha° - \beta°) = \frac{A}{B}\angle 0° = \frac{A}{B}$$

Terlihat bahwa ketika V dan I satu phasa, impedansi yang dihasilkan seluruhnya komponen riil atau impedansi kompleks hanya terdiri dari komponen resistor murni (R). Dengan kata lain konsep resonansi adalah menghilangkan komponen imaginer / reaktansi saling meniadakan.

Resonansi Seri

Impedansi total:

$$\mathbf{Z}_{tot} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

saat resonansi :

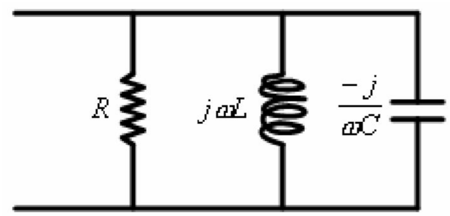
$$\omega L - \frac{1}{\omega C} = 0 \rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi **Z** minimum, sehingga arusnya maksimum.

Resonansi Paralel



Admitansi total :

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{-j/\omega C} = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

saat resonansi :

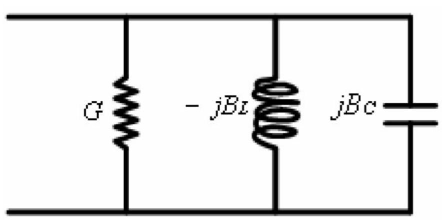
$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi **Z** maksimum, sehingga arusnya minimum.

Gambar tersebut dapat diganti notasinya :



Admitansi total :

$$Y = G + jB_C - jB_L$$

$$Y = G + j(\omega C - \frac{1}{\omega L})$$

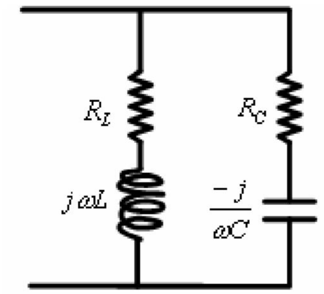
saat resonansi :

$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Resonansi Paralel 2 Cabang



$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right) + \frac{1}{R_C - \frac{j}{\omega C}}\left(\frac{R_C + \frac{j}{\omega C}}{R_C + \frac{j}{\omega C}}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L - j\omega L}{R_L^{\,2} + (\omega L)^2} + \frac{R_C + \frac{j}{\omega C}}{R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L}{R_L^{\,2} + (\omega L)^2} + \frac{R_C}{R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2} + j\left(\frac{\frac{1}{\omega C}}{R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{R_L^{\,2} + (\omega L)^2}\right)$$

saat resonansi:

$$\frac{\frac{1}{\omega C}}{R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{R_L^{\,2} + (\omega L)^2} = 0$$

$$\frac{\frac{1}{\omega C}}{R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2} = \frac{\omega L}{R_L^{\,2} + (\omega L)^2}$$

$$R_L^{\,2} + (\omega L)^2 = \omega^2 LC\left(R_C^{\,2} + \left(\frac{1}{\omega C}\right)^2\right)$$

$$R_L^{\,2} + \omega^2 L^2 = \omega^2 LCR_C^{\,2} + \frac{L}{C}$$

$$\omega^2 LCR_C^{\,2} - \omega^2 L^2 = R_L^{\,2} - \frac{L}{C}$$

$$\omega^2 LC\left(R_C^{\,2} - \frac{L}{C}\right) = R_L^{\,2} - \frac{L}{C}$$

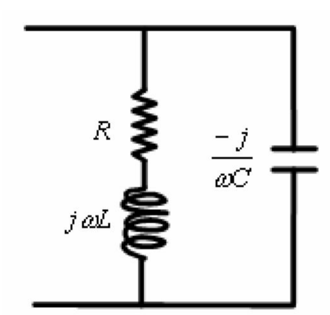
$$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L{}^2 - \frac{L}{C}}{R_C{}^2 - \frac{L}{C}}}$$

Perlu diingat bahwa : $\sqrt{\frac{R_L{}^2 - \frac{L}{C}}{R_C{}^2 - \frac{L}{C}}}$ harus positif real sehingga syarat :

$R_l{}^2 > \frac{L}{C}$ dan $R_C{}^2 > \frac{L}{C}$ atau $R_L{}^2 < \frac{L}{C}$ dan $R_C{}^2 < \frac{L}{C}$

Ketika nilai $R_L{}^2 = R_C{}^2 = \frac{L}{C}$, maka rangkaian beresonansi untuk semua frekuensi.

Resonansi Kombinasi 1



$$\mathbf{Z}_1 = R + j\omega L$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R + j\omega L} + j\omega C$$

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{1}{R + j\omega L}\left(\frac{R - j\omega L}{R - j\omega L}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

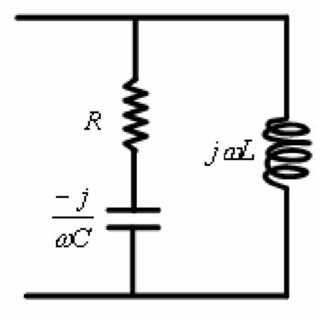
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

saat resonansi : $\omega_C = \frac{\omega_L}{R^2 + \omega^2 L^2}$, sehingga :

$$R^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \rightarrow \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \frac{1}{LC}\left(1 - \frac{R^2 C}{L^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2 C}{L^2}\right)}$$

Resonansi Kombinasi 2



$$\mathbf{Z}_1 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$\mathbf{Z}_2 = j\omega L$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{R - \frac{j}{\omega C}} + \frac{1}{j\omega L} = \frac{1}{R - \frac{j}{\omega C}} - \frac{j}{\omega L}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R - \frac{j}{\omega C}}\left(\frac{R + \frac{j}{\omega C}}{R + \frac{j}{\omega C}}\right) - \frac{j}{\omega L} = \frac{R + \frac{j}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} - \frac{j}{\omega L}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \frac{1}{\omega^2 C^2}} + j\left(\frac{\frac{1}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} - \frac{1}{\omega L}\right)$$

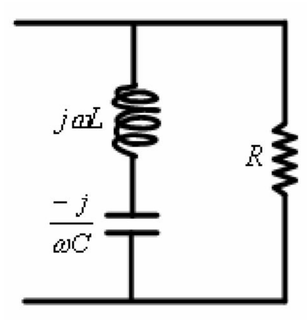
saat resonansi : $\dfrac{\frac{1}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} = \dfrac{1}{\omega L}$, sehingga :

$$R^2 + \frac{1}{\omega^2 C^2} = \frac{L}{C} \rightarrow \frac{1}{\omega^2 C^2} = \frac{L}{C} - R^2 \rightarrow \omega^2 C^2 = \frac{1}{\frac{L}{C} - R^2} \rightarrow \omega^2 = \frac{1}{C^2\left(\frac{L}{C} - R^2\right)}$$

$$\omega^2 = \frac{1}{LC - C^2 R^2} = \frac{1}{LC\left(1 - \frac{CR^2}{L}\right)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1 - \frac{CR^2}{L}\right)}}$$

Resonansi Kombinasi 3



$$\mathbf{Z}_1 = j\omega L + \frac{1}{j\omega C}$$

$$\mathbf{Z}_2 = R$$

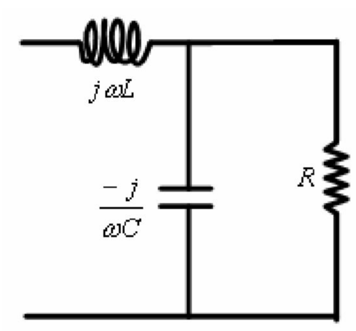
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{j\omega L + \frac{1}{j\omega C}} + \frac{1}{R} = \frac{1}{R} - \frac{j\omega C}{1-\omega^2 LC}$$

saat resonansi :

$$\frac{\omega C}{1-\omega^2 LC} = 0$$

$$fo = 0$$

Resonansi Kombinasi 4



$$\frac{1}{\mathbf{Z}_1} = \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R} + j\omega C \rightarrow \mathbf{Z}_1 = \frac{1}{\frac{1}{R} + j\omega C}$$

$$\mathbf{Z}_2 = j\omega L$$

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = \frac{1}{\frac{1}{R} + j\omega C} + j\omega L$$

$$\mathbf{Z}_{tot} = j\omega L + \frac{1}{\frac{1}{R} + j\omega C}\left(\frac{\frac{1}{R} - j\omega C}{\frac{1}{R} - j\omega C}\right) = j\omega L + \frac{\frac{1}{R} - j\omega C}{\frac{1}{R^2} + \omega^2 C^2}$$

$$\mathbf{Z}_{tot} = \frac{\frac{1}{R}}{\frac{1}{R^2}+\omega^2C^2} + j\left(\omega L - \frac{\omega C}{\frac{1}{R^2}+\omega^2C^2}\right)$$

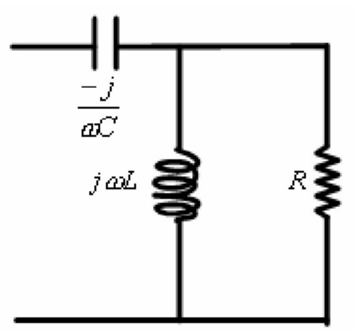
saat resonansi : $\omega L = \dfrac{\omega C}{\frac{1}{R^2}+\omega^2C^2}$, sehingga :

$$\frac{1}{R^2}+\omega^2C^2 = \frac{C}{L} \rightarrow \omega^2C^2 = \frac{C}{L}-\frac{1}{R^2}$$

$$\omega^2 = \frac{1}{C^2}\left(\frac{C}{L}-\frac{1}{R^2}\right) = \frac{1}{LC}-\frac{1}{C^2R^2} = \frac{1}{LC}\left(1-\frac{L}{CR^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{L}{CR^2}}$$

Resonansi Kombinasi 5



$$\frac{1}{\mathbf{Z}_1} = \frac{1}{R}+\frac{1}{j\omega L} = \frac{1}{R}-\frac{j}{\omega L} \rightarrow \mathbf{Z}_1 = \frac{1}{\frac{1}{R}-\frac{j}{\omega L}}$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$\mathbf{Z}_{tot} = \mathbf{Z}_1+\mathbf{Z}_2 = \frac{1}{\frac{1}{R}-\frac{j}{\omega L}} - \frac{j}{\omega C} = \frac{1}{\frac{1}{R}-\frac{j}{\omega L}}\left(\frac{\frac{1}{R}+\frac{j}{\omega L}}{\frac{1}{R}+\frac{j}{\omega L}}\right) - \frac{j}{\omega C}$$

$$\mathbf{Z}_{tot} = \frac{\frac{1}{R}+\frac{j}{\omega L}}{\frac{1}{R^2}+\frac{1}{\omega^2L^2}} - \frac{j}{\omega C} = \frac{\frac{1}{R}}{\frac{1}{R^2}+\frac{1}{\omega^2L^2}} + j\left(\frac{\frac{1}{\omega L}}{\frac{1}{R^2}+\frac{1}{\omega^2L^2}} - \frac{1}{\omega C}\right)$$

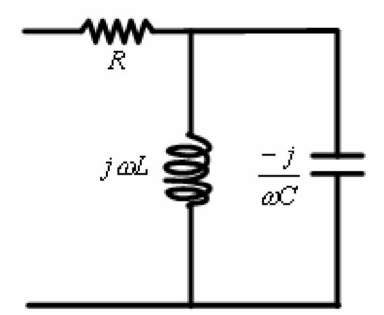
saat resonansi : $\dfrac{\dfrac{1}{\omega L}}{\dfrac{1}{R^2}+\dfrac{1}{\omega^2 L^2}}=\dfrac{1}{\omega C}$, sehingga :

$$\frac{1}{R^2}+\frac{1}{\omega^2 L^2}=\frac{C}{L}\rightarrow\frac{1}{\omega^2 L^2}=\frac{C}{L}-\frac{1}{R^2}\rightarrow\omega^2 L^2=\frac{1}{\dfrac{C}{L}-\dfrac{1}{R^2}}$$

$$\omega^2=\frac{1}{L^2\left(\dfrac{C}{L}-\dfrac{1}{R^2}\right)}=\frac{1}{LC\left(1-\dfrac{L}{CR^2}\right)}$$

$$f_0=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1-\dfrac{L}{CR^2}}}$$

Resonansi Kombinasi 6



$$\frac{1}{\mathbf{Z}_1}=\frac{1}{j\omega L}+\frac{1}{\dfrac{1}{j\omega C}}=\frac{1}{j\omega L}+j\omega C\rightarrow\mathbf{Z}_1=\frac{1}{\dfrac{1}{j\omega L}+j\omega C}$$

$$\mathbf{Z}_2=R$$

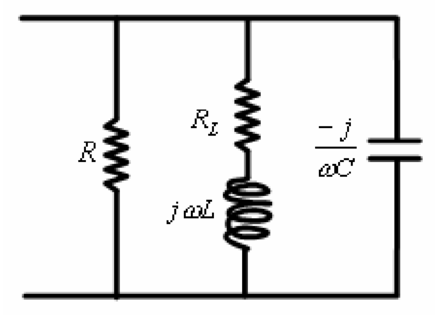
$$\mathbf{Z}_{tot}=\mathbf{Z}_1+\mathbf{Z}_2=\frac{1}{\dfrac{1}{j\omega L}+j\omega C}+R=R+\frac{j\omega L}{1-\omega^2 LC}$$

saat resonansi :

$$\frac{\omega L}{1-\omega^2 LC}=0$$

$$fo=0$$

Resonansi Paralel 3 Cabang



$$\mathbf{Z}_1 = R_L + j\omega L$$
$$\mathbf{Z}_2 = R$$
$$\mathbf{Z}_3 = \frac{1}{j\omega C}$$
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{R_L + j\omega L} + \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}}$$
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\omega C + \frac{1}{R_L + j\omega L} = \frac{1}{R} + j\omega C + \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right)$$
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\omega C + \frac{R_L - j\omega L}{R_L{}^2 + \omega^2 L^2} = \frac{1}{R} + \frac{R_L}{R_L{}^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R_L{}^2 + \omega^2 L^2}\right)$$

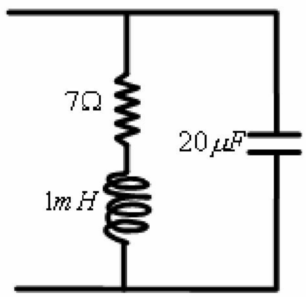
saat resonansi : $\omega C = \dfrac{\omega L}{R_L{}^2 - \omega^2 L^2}$, sehingga :

$$R_L{}^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R_L{}^2$$
$$\omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R_L{}^2\right) = \frac{1}{LC} - \frac{R_L{}^2}{L^2} = \frac{1}{LC}\left(1 - \frac{CR_L{}^2}{L}\right)$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR_L{}^2}{L}}$$

Contoh latihan :

1. Suatu rangkaian seri RLC dengan $R = 50\Omega, L = 0{,}05H, C = 20\mu F$ terpasang pada $V = 100\angle 0^o$ dengan frekuensi variabel. Pada frekuensi berapa tegangan induktor mencapai maksimum ? Berapakah tegangan induktor tersebut ?

2. Tentukan frekuensi resonansi pada gambat berikut :



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