# Fluid Statics

# Outline

- Definisi statis fluida
- Keseimbangan hidrostatik
- Persamaan barometrik
- Pengukuran tekanan: perangkat
- Prinsip dan gaya apung

# Learning Outcome

Pada akhir bab ini, Anda harus dapat:

- Mendefinisikan istilah "statis fluida."
- Menghitung tekanan menggunakan manometer.
- Menentukan gaya apung.
- Menerapkan prinsip statis fluida dalam operasi teknik kimia.

# What Is Fluid Statics?

**Apa Itu Fluida Statis ?**

Fluida Statis (juga disebut hidrostatika) adalah ilmu yang mempelajari gaya yang diterapkan oleh fluida dalam keadaan diam atau dalam gerak tubuh yang seragam.

**Karakteristik:**

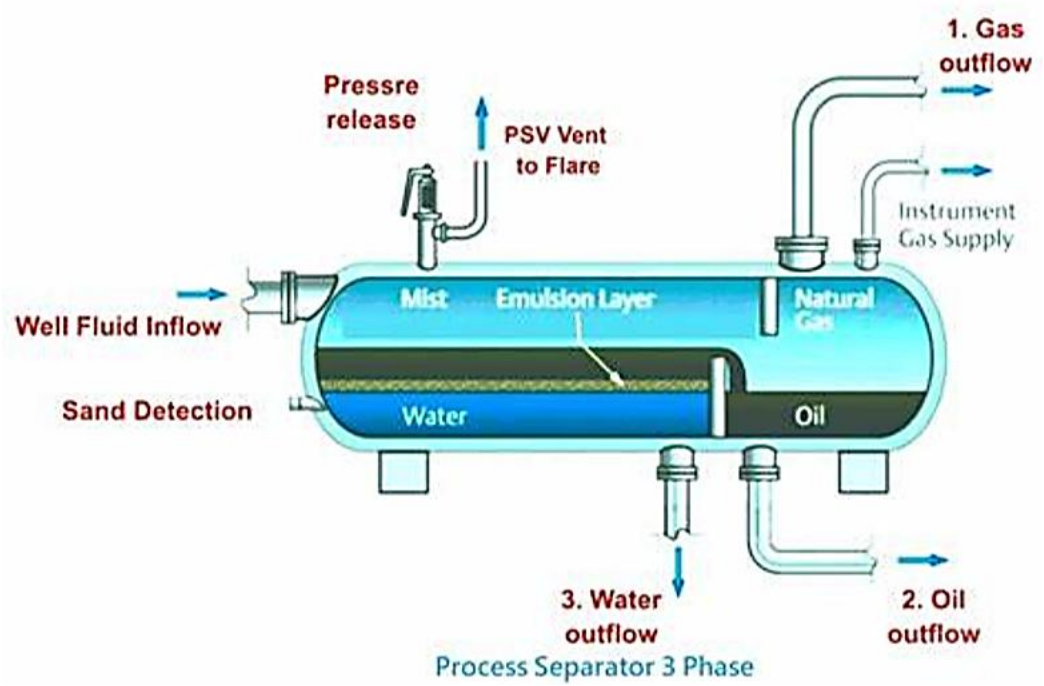
- Tidak ada tegangan geser yang terlibat.
- Satu-satunya gaya yang bekerja pada permukaan partikel adalah akibat **tekanan.**

**Aplikasi Statis Fluida**

1. Manometer
2. Dekanter gravitasi kontinu
3. Dekanter sentrifugal
4. Penentuan gaya apung
5. Aplikasi lainnya (misalnya gerak benda tegar dalam fluida)

## Fluid Statics

- Example:
  - Continuous gravity decanter



## Fluid Statics

- Example:
  - Water in a tank.
  - Water in a lake (Water actually move very slowly in the lake. However the movement of water relative to each other is nearly zero that water is seen as "static")

# Pressure

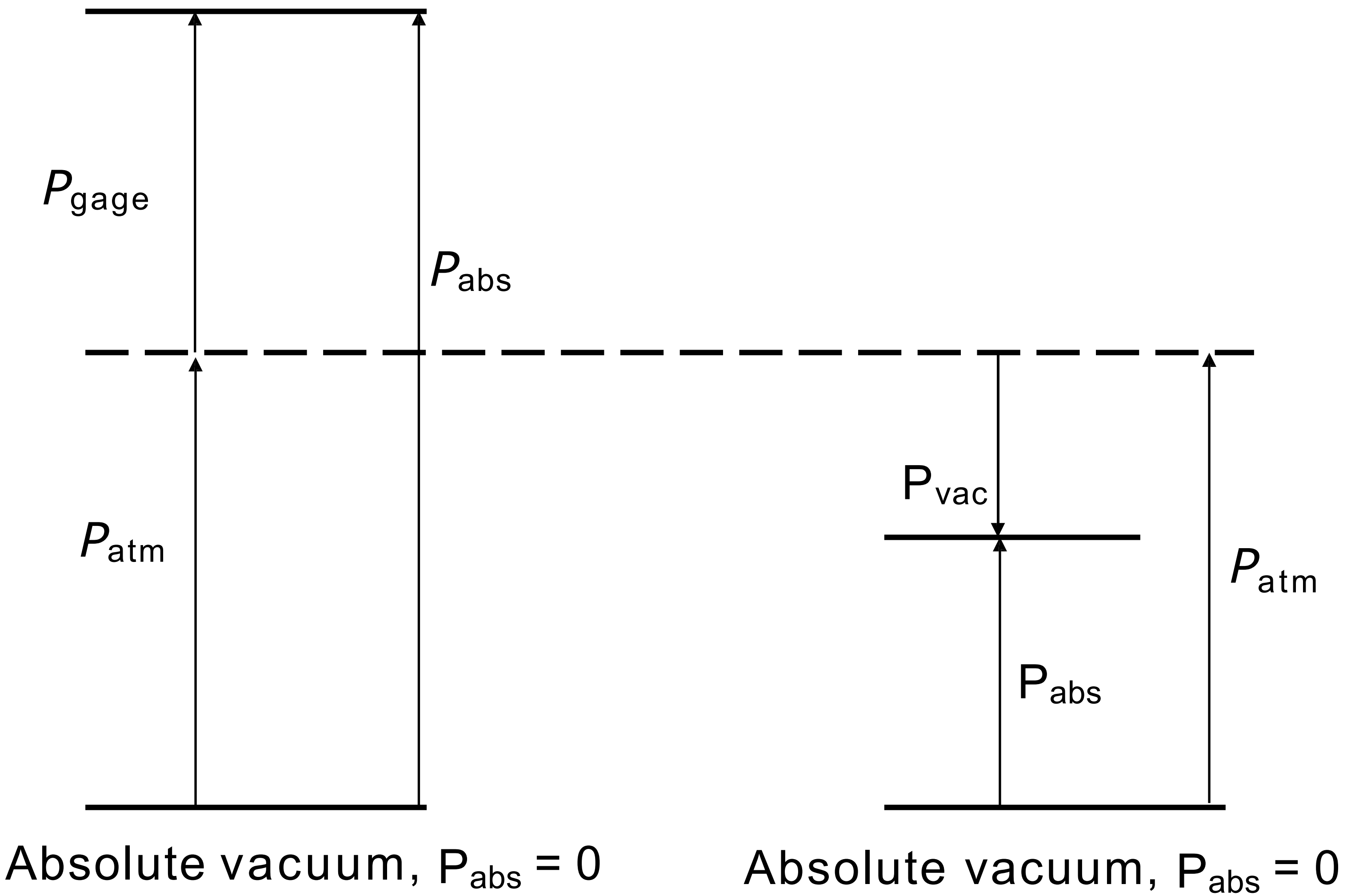
**Definisi:** Tekanan adalah gaya normal yang diberikan oleh fluida per satuan luas.

**Satuan:** Newton per meter persegi ($N/m^2$) atau Pascal (Pa).

Jenis tekanan dalam mekanika fluida:

- **Tekanan absolut (P abs):** Tekanan aktual di suatu posisi.
- **Tekanan gauge (P gauge):** P abs – P atm.
- **Tekanan vakum (P vac):** P atm – P abs.

# Absolute, Gage and Vacuum pressures




$$P_{abs} = P_{atm} + P_{gage}$$

$$P_{abs} = P_{atm} - P_{vac}$$

# Atmospheric Pressure

- Tekanan atmosfer tertinggi: 108.6 kPa (Tosontsengel, Mongolia, 19 Desember 2001).
- Tekanan atmosfer terendah non-tornadis: 87.0 kPa (Samudra Pasifik Barat, saat Topan Tip, 12 Oktober 1979).
- Standar dalam mekanika fluida: P atm = 101.325 kPa = 1 atm = 14.7 psi.

**Class Example 1**

a) A vacuum gage connected to a chamber reads 24 kPa at location where the local atmospheric pressure is 92 kPa. Determine $P_{abs}$.

b) If the absolute pressure in a tank is 20 psi at normal atmospheric pressure, determine $P_{gage}$.

c) Gage pressure readings shows a value of $-60$ kPa. What does it means? And, determine $P_{abs}$.

# Class Example 1: Solution

a) $P_{gauge} = 24$ kPa, $P_{atm} = 92$ kPa

$P_{abs} = 24 + 92 = 116$ kPa

b) $P_{gauge} = P_{abs} - P_{atm}$

$= 20 - 14.7$

$= 5.3$ psi

c) $P_{gage} = -60$ kPa

It means vacuum since the value is negative.

(The same if mention as $P_{vac} = 60$ kPa)

$P_{abs} = P_{gauge} + P_{atm}$

$= 101.3 - 60 = 41.3$ kPa

# Pressure Variation in a Fluid at Rest

**Hukum Pascal:**
•Tekanan pada satu titik dalam fluida sama dalam semua arah.
•Pada fluida diam, tekanan di bidang horizontal adalah sama.

**Keseimbangan Gaya dalam Fluida Diam:**
•Karena fluida dalam keadaan diam, tidak ada percepatan.
•Hukum Newton menyatakan bahwa jumlah gaya pada bagian mana pun dari fluida harus nol.

**Analisis dalam Arah Z (Vertikal):**
•Dipilih elemen fluida berbentuk kubus kecil dengan ketebalan $\Delta z$.
•Tekanan bekerja pada bagian atas ($z = \Delta z$) dan bagian bawah ($z = 0$) dari elemen fluida.
•Gaya berat juga bekerja ke bawah.

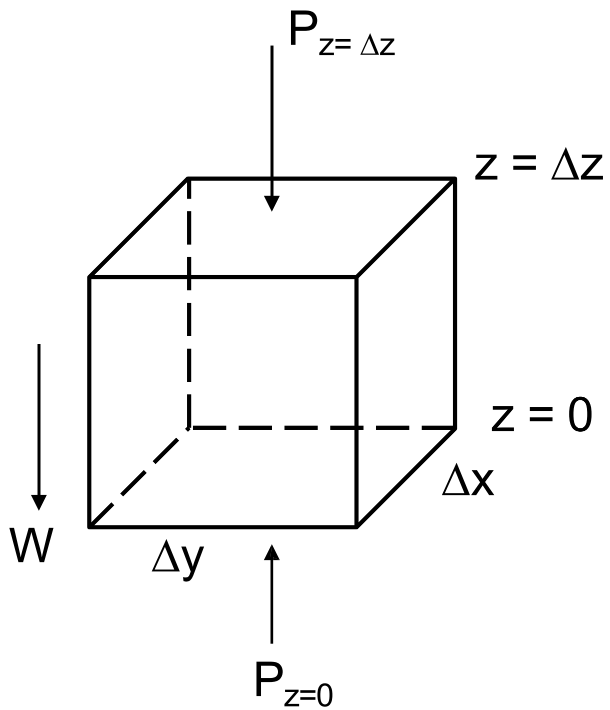



***Fig.*** Surface and body forces acting on small fluid element.

$$F_{bottom} - F_{top} - F_{weight} = 0$$

We can write,

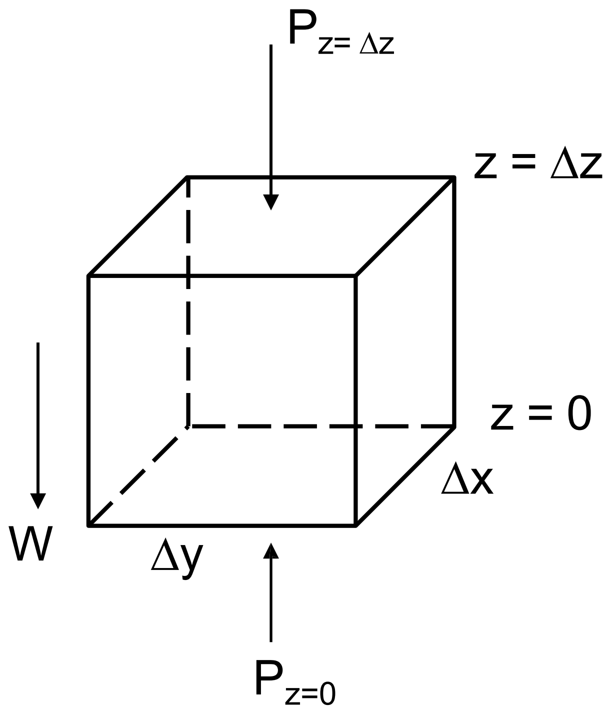
$$(P_{z=0})\, \Delta x\, \Delta y - (P_{z=\Delta z})\, \Delta x\, \Delta y - \rho g\, \Delta x\, \Delta y\, \Delta z = 0 \quad \ldots\ldots\ldots(1)$$

Dividing equation 1 by $\Delta x\ \Delta y\ \Delta z$ and rearranging, we find

$$\frac{P_{z=\Delta z} - P_{z=0}}{\Delta z} = -\rho g$$

If we now let $\Delta z$ approach zero, then

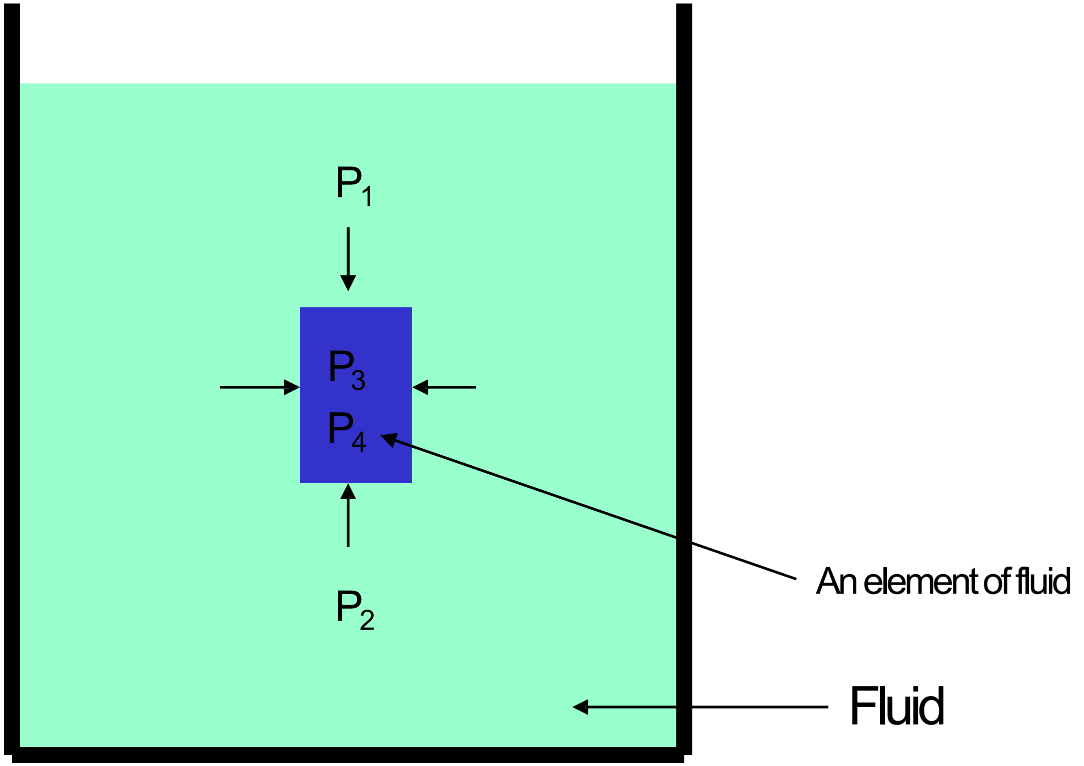
$$\lim_{\Delta z \to 0} \frac{\Delta P}{\Delta z} = \frac{dP}{dz} = -\rho g = -\gamma$$




This is the basic equation of fluid statics, called the **Barometric equation**

# Pressure Variation in a Fluid at Rest

- Jika terdapat sebuah benda yang melayang dalam fluida yang diam, tekanan diberikan secara sama ke segala arah dan disebut tekanan statis.



# Persamaan Barometrik

- Persamaan ini menyatakan hubungan antara tekanan dan ketinggian dalam fluida diam:

$$dP = -\rho g dz$$

- P adalah tekanan,
- Ρ adalah berat jenis fluida,
- dz adalah perubahan ketinggian.

Untuk Fluida inkompresible

$$P2 - P1 = \rho\ g\ dz$$

## Pressure- depth relationships for incompressible fluids

### What is incompressible fluids??

- Fluida inkompresibel adalah fluida yang tidak mengalami perubahan volume (atau massa jenis) akibat perubahan tekanan eksternal.
- Kompressibilitas adalah ukuran seberapa besar perubahan volume fluida terhadap tekanan eksternal.
- Dalam fluida inkompresibel, kompresibilitas dianggap nol, yang berarti massa jenis tetap konstan di seluruh aliran.

Example: water

What is the difference between Compressible Fluids and Incompressible Fluids ?

# Pressure- depth relationships for incompressible fluids

*Assumption*: g is constant = 9,8 m/s$^2$

We can write barometric equation can be directly integrated

$$\int_1^2 dP = -\int_1^2 \rho g \, dz$$

to yield

$$= -\rho g \int_1^2 dz$$

$$P_2 - P_1 = -\rho g \, (z_2 - z_1)$$

or $$P_2 - P_1 = \rho g \, (z_1 - z_2)$$

or $$P_2 = P_1 + \rho g \, h$$

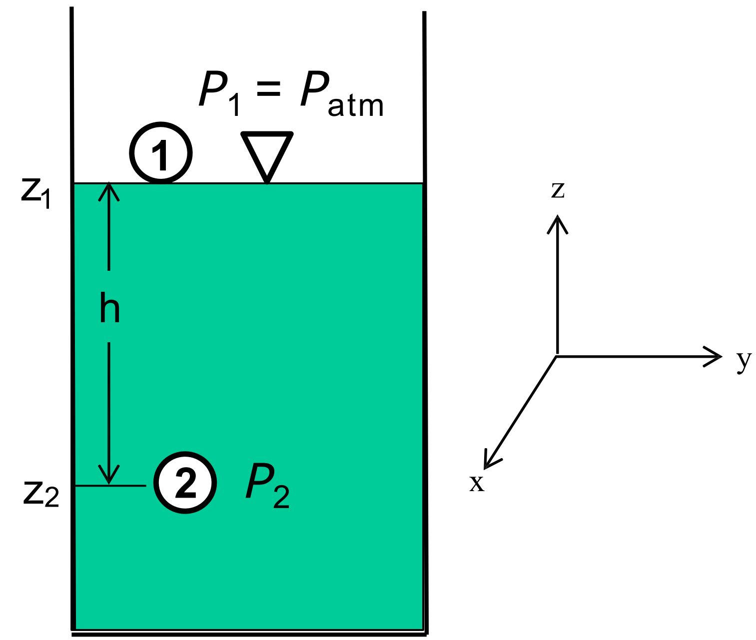



***Fig.*** Notation for pressure variation in a fluid at rest with a free surface

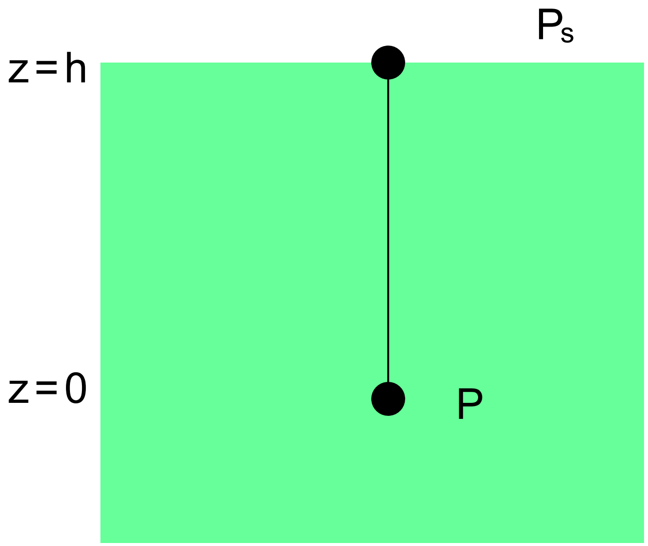
**Pressure in a liquid at rest increases linearly with distance from the free surface**

# Pressure- depth relationships for incompressible fluids

$$\int_{P}^{P_s} dP = -\int_{0}^{h} \gamma \, dz$$

$$P_s - P = -\gamma h$$

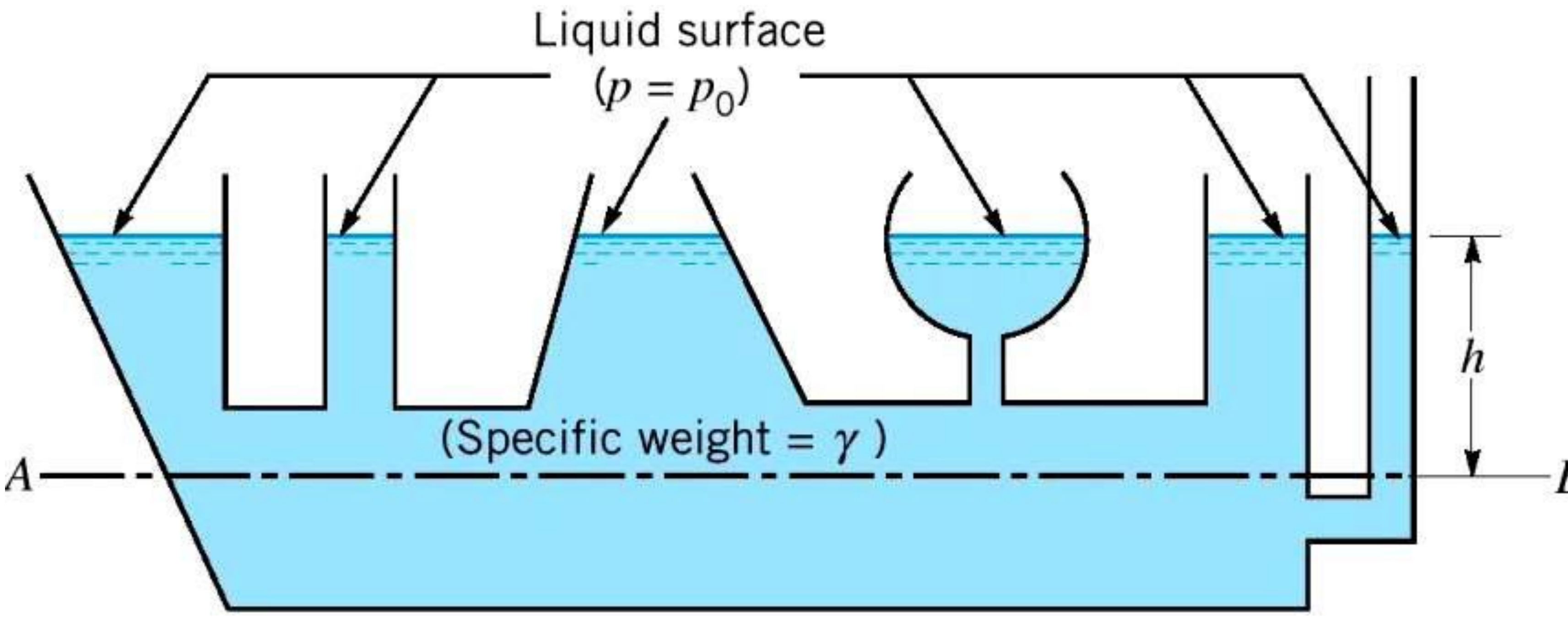
$$P = P_s + \gamma h$$




- For free surface → $P_s = P_{atm}$
- Normally take $P_s = 0$ ($P_{gage}$ definition)
- $P = \gamma h = P_{gage}$

- P increase as one go downward.
- P decrease as one go upward.

# Pressure at the same elevation




At same elevation, pressure is the same for the same fluid at rest.

## Class Example 2

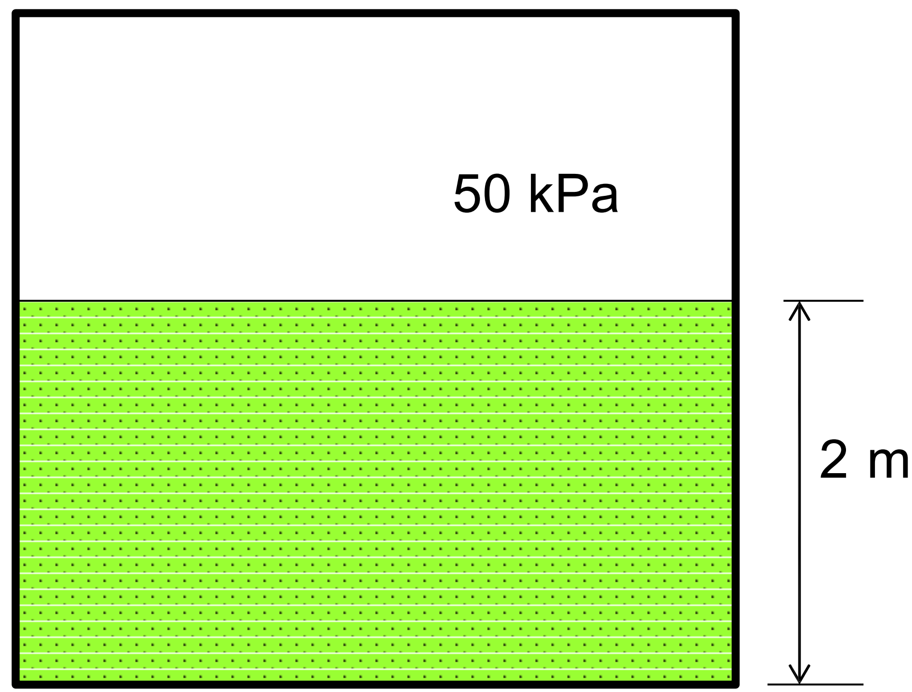
Find the pressure at the bottom of a tank containing glycerin under pressure shown

($\rho g_{\text{ glycerin}}$ = 12.34 N/m³)

***Solution:***

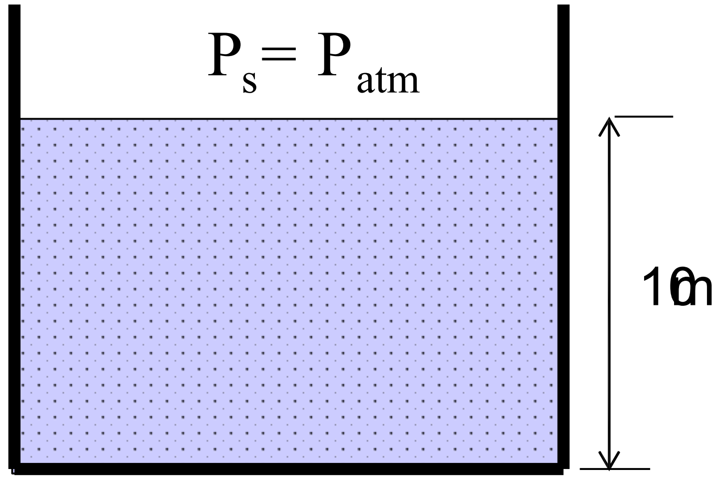
*Assumption*: glycerin is an incompressible fluid

$$P_{bottom} = P_s + \rho g\, h$$
$$= 50 + (12.34)(2)$$
$$= 74.68 \text{ kPa}$$



## Class Example 3

Determine the pressure at the bottom of an open tank containing water at atmospheric pressure.




*Solution:*

***Assumption:***

water is an incompressible fluid

$$P_{bottom} = P_s + \rho g h$$

$$= Ps + (1000)(9.81)(10)$$

If we measure the pressure relative to atmospheric pressure (gage pressure),It follows that Ps = 0, therefore,

$$\mathbf{P_{bottom} = 98.1\ kPa\ (gage)}$$

# Compressible Fluid

- Density ρ change with pressure
  Example; Ideal/perfect gases

The equation of state for ideal gas is

$$\rho = \frac{P}{RT} = \frac{PM}{R_u T}$$

P = absulate pressure (kPa)
M = molar mass (kg/kmol)
$R_u$ = Universal gas constant (J/mol K)
R = Gas constant (kJ/kg K)
T = absolute temperature (K)

Substituting above equation for density in Barometric equation, we find

$$\frac{dP}{dz} = -\frac{PM}{R_u T} g \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If the temperature is constant, equation 1 can be separated and integrated as follows:

$$\int_1^2 \frac{dP}{P} = -\frac{gM}{R_u T}\int_1^2 dz$$

$$\ln \frac{P_2}{P_1} = -\frac{gM_w}{R_u T}(z_2 - z_1)$$

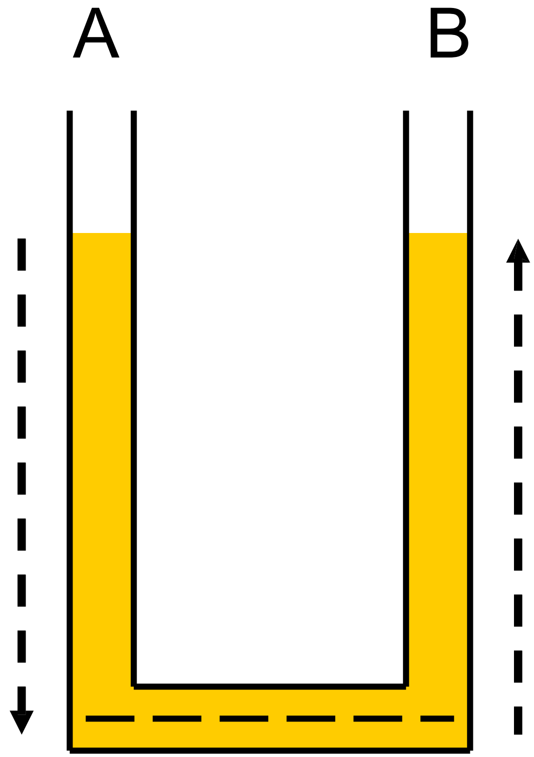
$$P_2 = P_1 \exp\left(-\frac{gM_w}{R_u T}(z_2 - z_1)\right)$$

## Pressure Measurement

- Objective:
  - Understand the principles of manometer
  - Learn how to calculate pressure using manometer

# In measuring pressure:

- Pressure increases as one go downward
- In calculation, has to "plus" the pressure



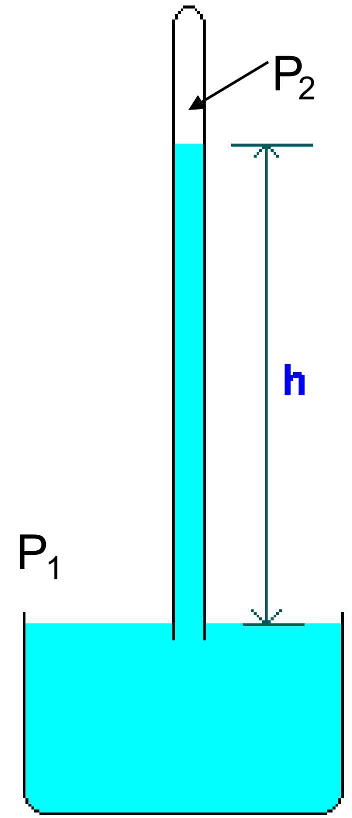

- Pressure decreases as one go upward
- In calculation, has to "minus" the pressure

# Measurement of Pressure

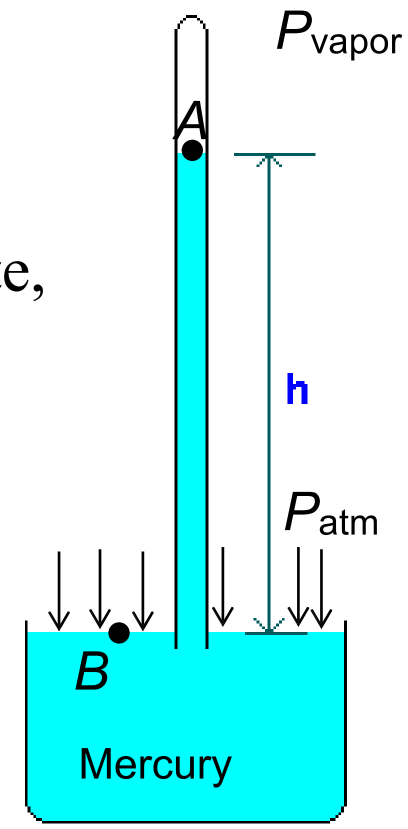
## Barometer : (Mercury Barometer)

A barometer is used to measure atmospheric pressure

- A simple barometer consists of:
  - a tube more than 30 inch (760 mm) long
  - inserted in an open container of mercury
  - a closed and evacuated end pointing upward
  - open end in the mercury pool
  - mercury extending from the container up into the tube.
  - It contains mercury vapor at its saturated vapor pressure



# Measurement of atmospheric pressure




In reference to barometric equation, we can write,

$$P_{atm} = \gamma_{Hg} h + P_{vapor}$$

$$P\ atm = \rho\ g\ h + P\ vapor$$

Since pressure exerted by mercury vapor is very small, therefore,

$$P_{atm} = \gamma_{Hg} h$$

**Manometer:** They use vertical or inclined liquid columns to measure pressure

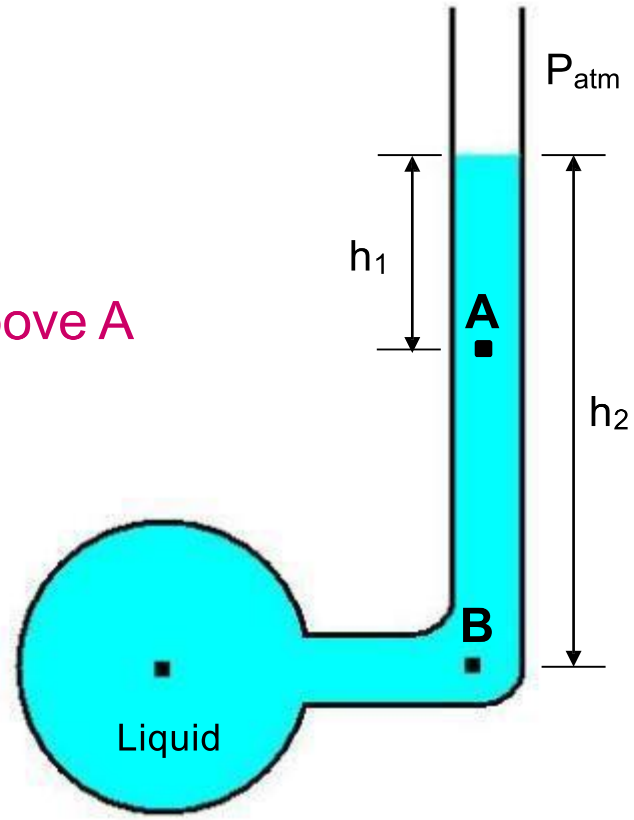
A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes.

- Piezometer tube
- U-tube manometer
- Inverted U-tube manometer
- Inclined tube manometer

# Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured.

As the tube is open to the atmosphere the pressure is relative to atmospheric so is gauge pressure




Pressure at A = pressure due to column of liquid above A

$$P_A = P_{atm} + \rho g h_1 = \rho g h_1$$

($P_{atm}$ → 0)

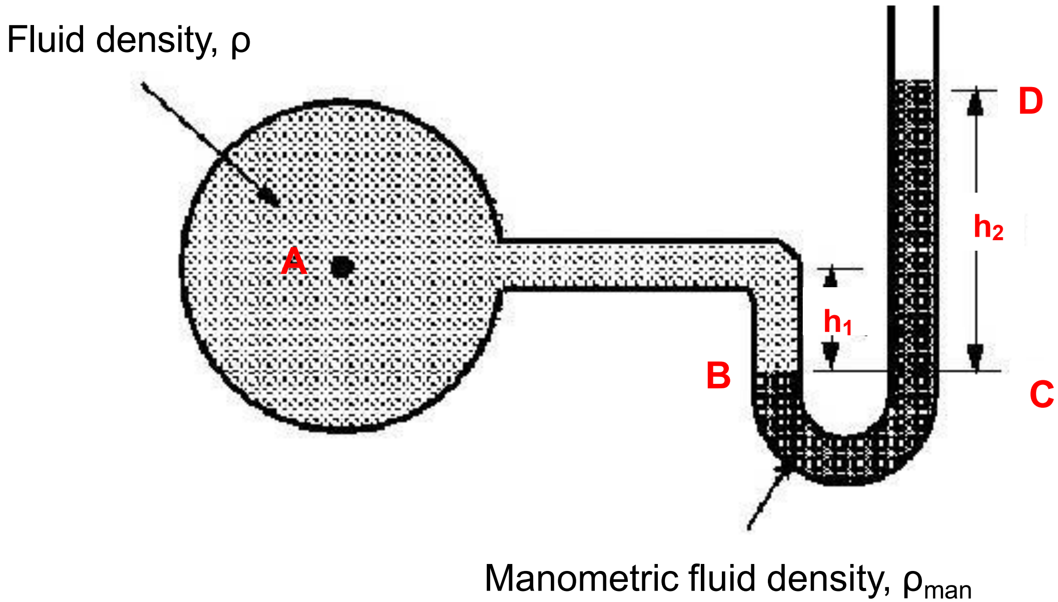
Pressure at B = pressure due to column of liquid above B

$$P_B = P_{atm} + \rho g h_2 = 0 + \rho g h_2$$

This method can only be used for liquids (*i.e.*, **not** for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable

# The "U"-Tube Manometer

Using a "U"-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The "U" is connected as in the figure below and filled with a fluid called the manometric fluid. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



Pressure in a continuous static fluid is the same at any horizontal level so,

$$P_B = P_C \quad \ldots\ldots\ldots.(1)$$

For the **left hand** arm,

Pressure at B = pressure at A + pressure due to height $h_1$ of measured fluid

$$P_B = P_A + \rho g h_1$$

For the **right hand** arm

Pressure at C = pressure at D + pressure due to height h of manometric fluid

$$P_C = P_{atm} + \rho_{man} g h_2$$

$= \rho_{man} g h_2$ ( Since we are measuring gage pressure, $P_{atm} = 0$ )

Putting the values of $P_B$ and $P_C$ in equation 1, we find, $\mathbf{P_A = \rho_{man} g h_2 - \rho g h_1}$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid *i.e.*, $\rho_{man} >> \rho$. In this case the term $\rho g h_1$ can be neglected, and the gauge pressure give by

$$\mathbf{P_A = \rho_{man} g h_2}$$

## Measurement of Pressure Difference Using a "U"-Tube Manometer

If the "U"-tube manometer is connected to a pressurized vessel at two points the pressure difference between these two points can be measured.

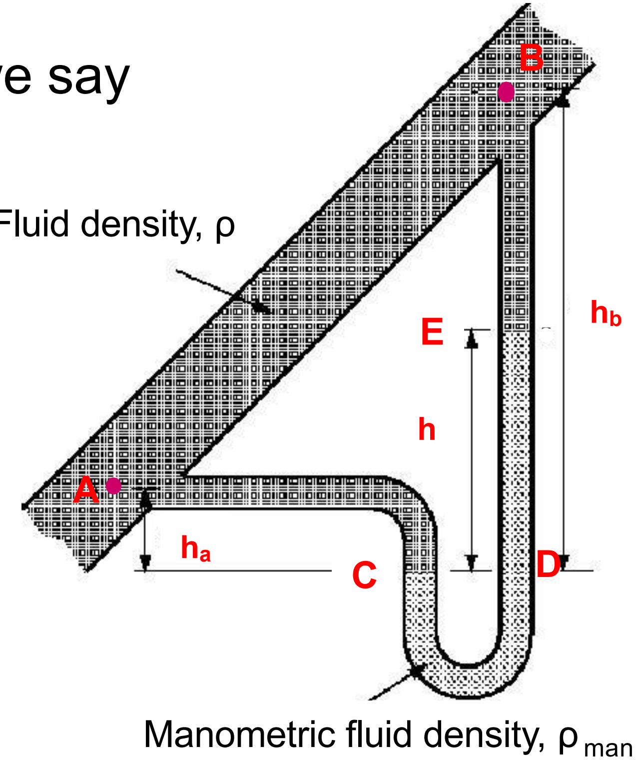
If the manometer is arranged as in the figure, then we say

Pressure at C = Pressure at D

$$P_C = P_D \quad .........(1)$$

$$P_C = P_A + \rho g h_a \; ....... \; (2)$$

$$P_D = P_B + \rho g(h_b - h) + \rho_{man} g h \; ......(3)$$




Combining equation 1, 2 & 3,

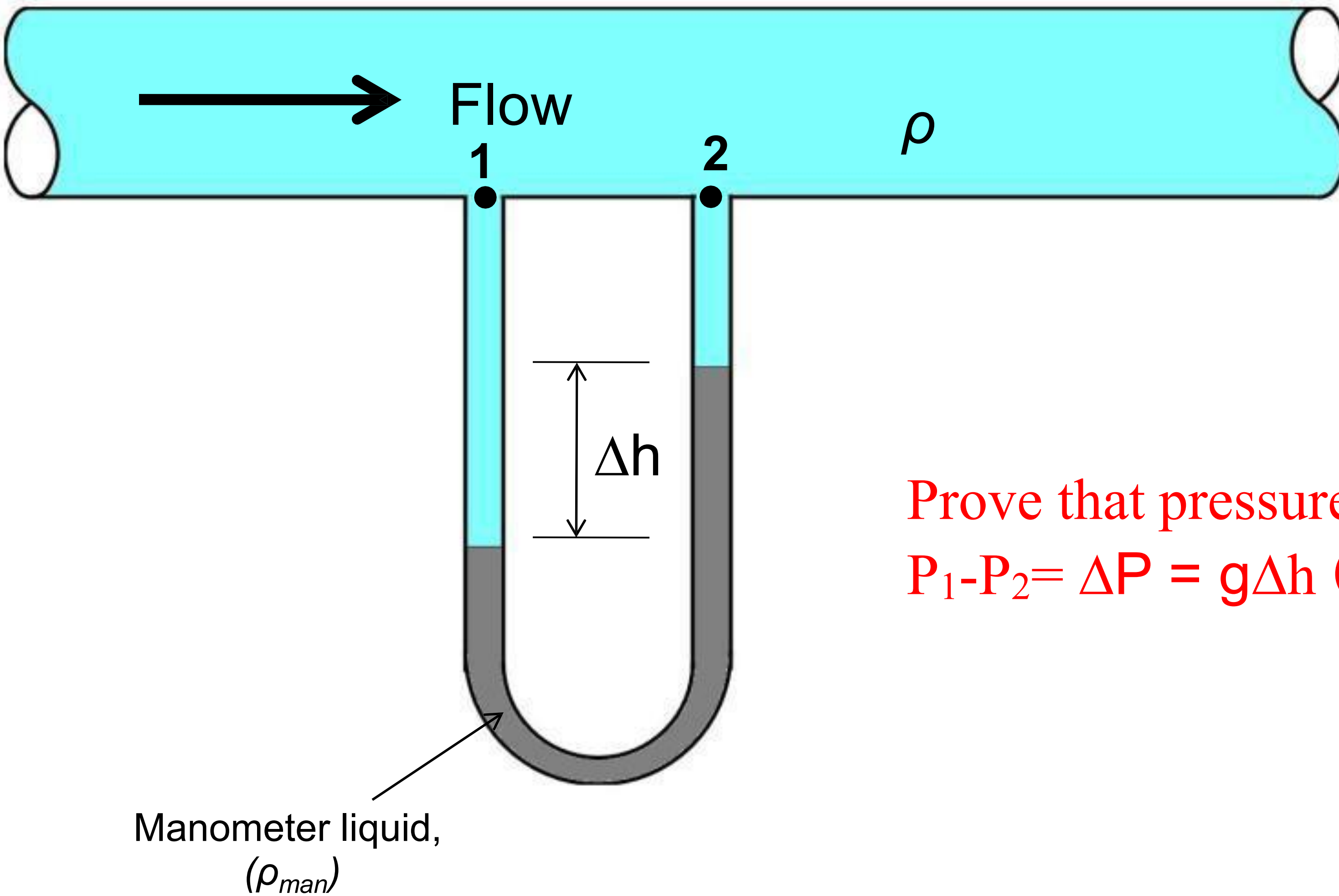
$$P_A + \rho g h_a = P_B + \rho g(h_b - h) + \rho_{man} g h$$

or, $P_A - P_B = \rho g(h_b - h_a) + (\rho_{man} - \rho) g h$

if the fluid whose pressure difference is being measured is a gas and $\rho_{man} >> \rho$, then the terms involving ρ can be neglected, so

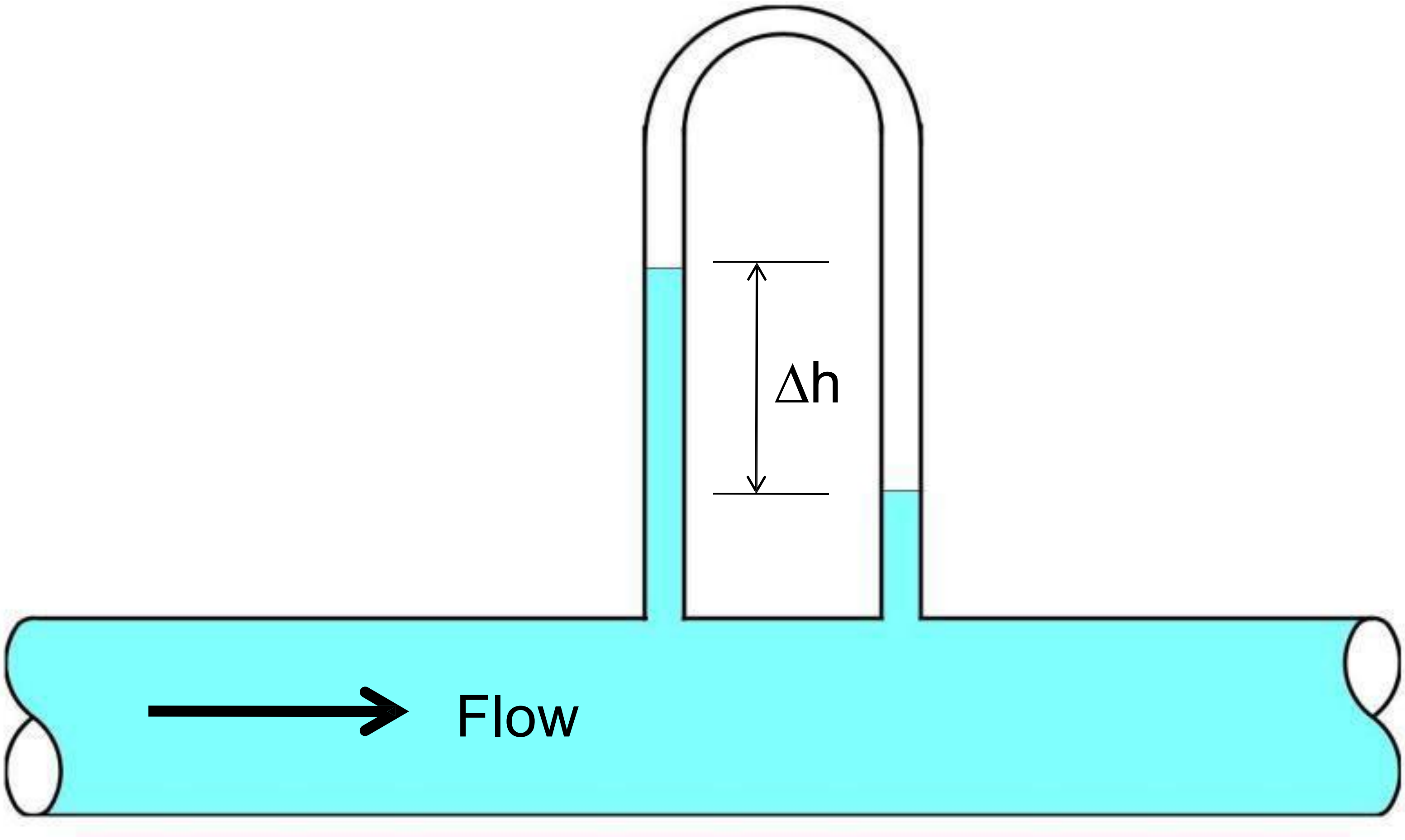
$$P_A - P_B = \rho_{man} g h$$

At the lower part of the pipe, require another manometer liquid to measure $\Delta P$



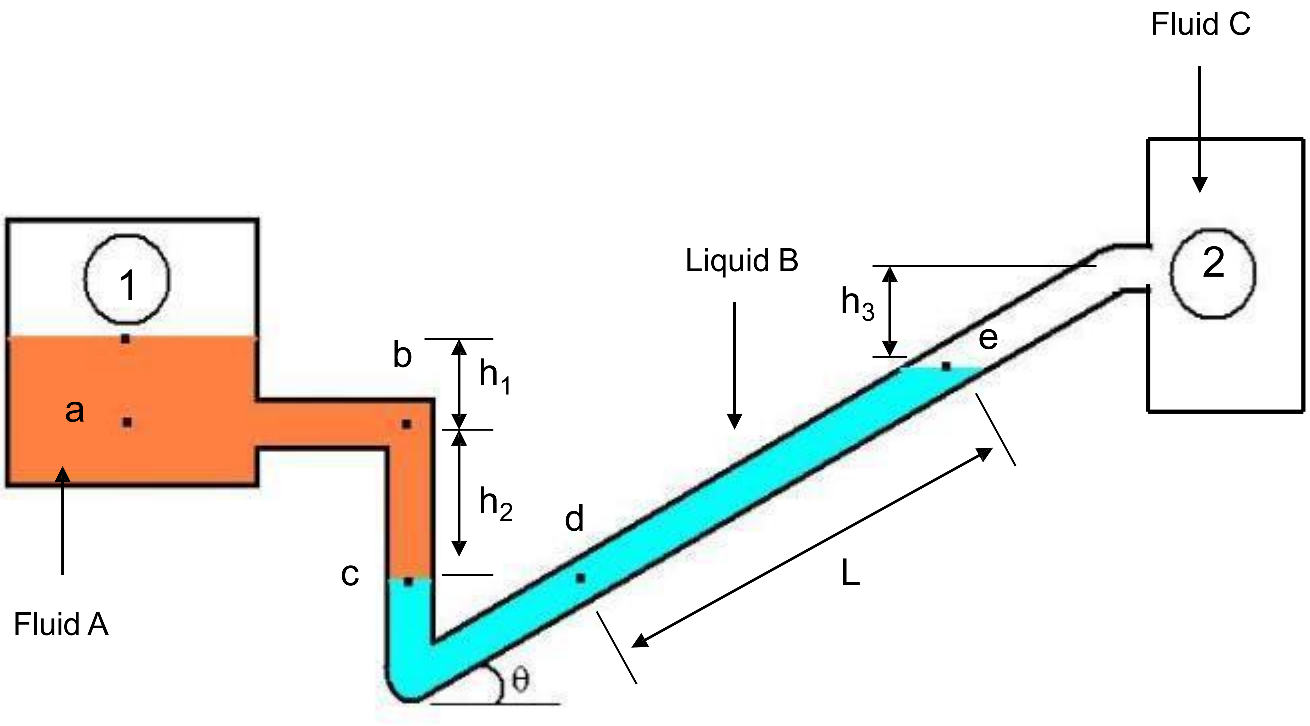

Prove that pressure difference

$P_1 - P_2 = \Delta P = g\Delta h\,(\rho_{man} - \rho)$




At the upper part of the pipe,it does not require another manometer liquid to measure ΔP

# Inclined-tube manometer




**Prove that $P_1 - P_2 = \gamma_B L \sin\theta$**

## Selection of Manometer

Care must be taken when attaching the manometer to vessel, no burrs must be present around this joint. Burrs would alter the flow causing local pressure variations to affect the measurement.

## Advantages and disadvanges of Manometer

**Advantages**

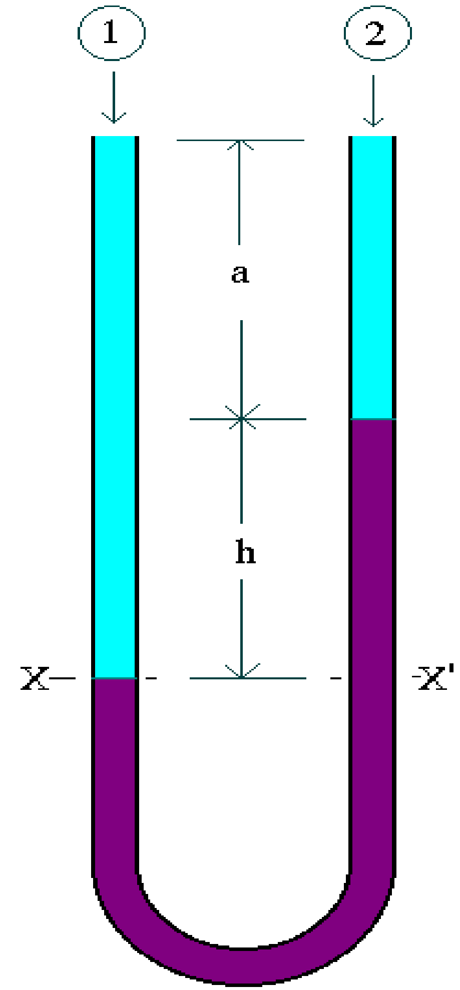
- small pressure differences can be measured
- They are very simple
- No calibration is required; the pressure difference can be calculated from first principles.

**Disadvantages**

- Not for measuring larger pressure differences
- Some liquids are unsuitable for use. Surface tension can also cause errors due to capillary rise; this can be avoided if the diameters of the tubes are sufficiently large - preferably not less than 15 mm diameter
- Slow response; unsuitable for measuring fluctuating pressures.

# Class example 1

A simple U-tube manometer is installed across an orifice meter. The manometer is filled with mercury (specific gravity 13.6), and the liquid above the mercury is carbon tetrachloride (specific gravity 1.6). The manometer reads 200 mm. What is the pressure difference over the manometer?




**Solution:**

**Pressure at X = pressure at X'**

Pressure at X, $P_X = P_1 + \rho_{CCl4} g\,(a+h)$

Pressure at X', $P_{X'} = P_2 + \rho_{mer} g\,h + \rho_{CCl4} g\,a$

**We can write,**

$P_1 + \rho_{CCl4} g\,(a+h) = P_2 + \rho_{mer} g\,h + \rho_{CCl4} g\,a$

or, $P_1 - P_2 = \rho_{mer} g\,h - \rho_{CCl4} g\,h$

$= gh\rho_{water}\,(SG_{mercury} - SG_{CCl4})$

$= 9.81 \times 0.2 \times 1000\,(13.6 - 1.6)$

$= 23544$ Pa

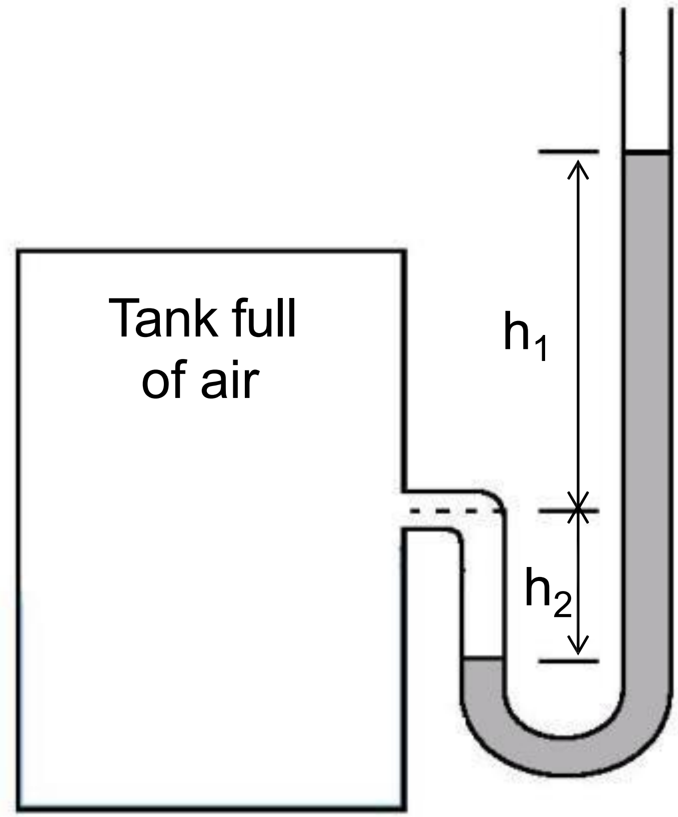
**The pressure difference is 23544 Pa**

# Class example 2

The fluid shown shaded in the manometer is ethyl iodide with a specific gravity of 1.93. The heights are $h_1$ = 100 cm and $h_2$ = 20 cm.

a) What is the gage pressure in the tank?

b) What is the absolute pressure in the tank?



$$P_{tank} + \rho_{air} g h_2 - \rho_{El} g(h_1 + h_2) = P_{atm}$$

Since we're taking gage pressure, $P_{atm} = 0$

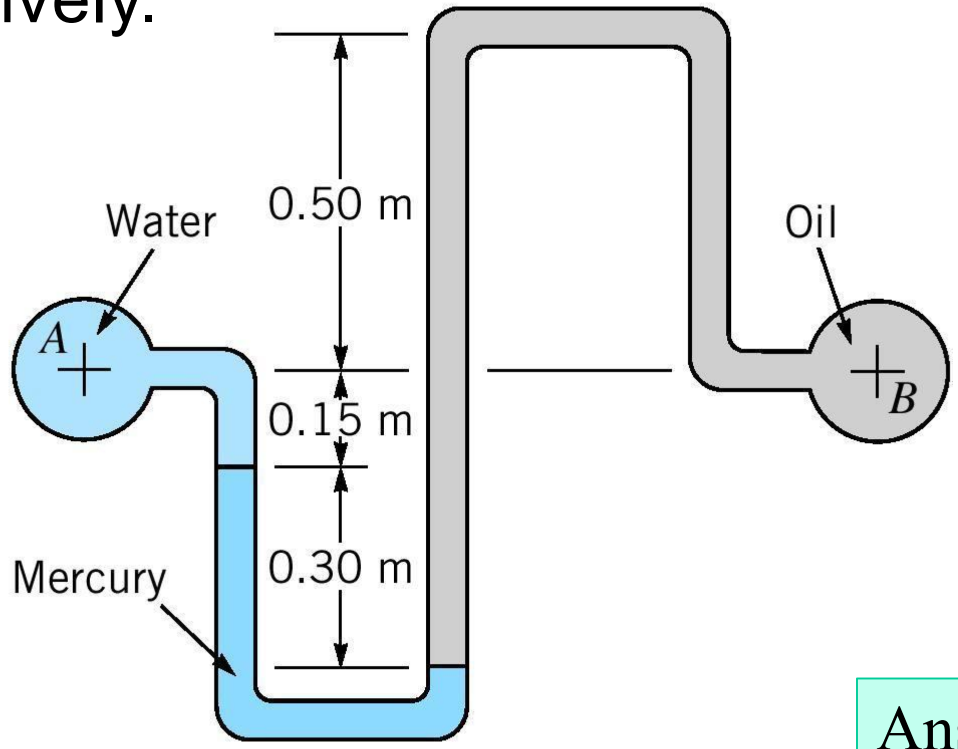
$$P_{tank} = \rho_{air} g h_2 + \rho_{El} g(h_1 + h_2)$$
$$= -(1.2)(9.81)(0.2) + (1.93)(1000)(9.81)(1.00 + 0.200)$$
$$= 22717.6 \text{ Pa}$$
$$= 22.72 \text{ kPag}$$

Assuming the density of air is too small

$$P_{tank} = \rho_{air} g h_2 + \rho_{El} g(h_1 + h_2)$$
$$= (1.93)(1000)(9.81)(1.00 + 0.200)$$
$$= 22719.96 \text{ Pa}$$
$$= 22.72 \text{ kPag}$$

## Self assessment Assignment

- The mercury manometer below indicates a differential reading of 0.30 m when the pressure in pipe A is 30-mm Hg vacuum. Determine the pressure in pipe B if the specific gravity of the oil and mercury is 0.91 and 13.6, respectively.




Ans: 33.47 kPa

## Mechanical and Electronic Pressure Measuring Device

- Measure of high pressures

- Pressure gage

  Used where only a visual indication is needed at the site where the pressure is being measured.

- Pressure transducer

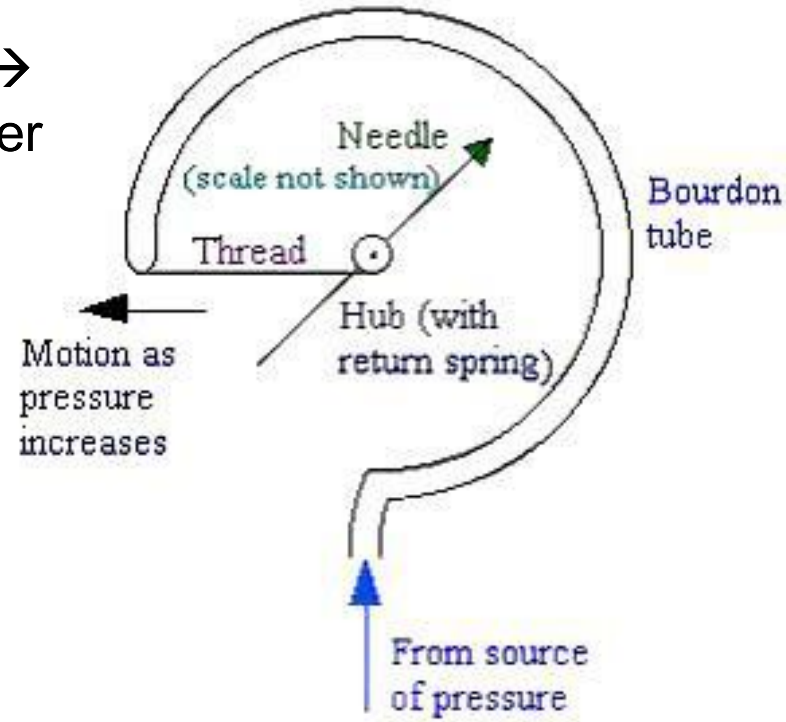
  Pressure is at a point, and the measured value is another point. displayed at

# Bourdon Pressure Gage

- When pressure acts on an elastic structure, the structure will deform.
- This deformation relates to the magnitude of pressure
- As the pressure within tube increases → tube straighten → translated into pointer motion on dial.
- The pressure indicated

  → the difference between that communicated by the system to the external (ambient) pressure

  → $P_{gage}$



## Pressure transducer

- The sensed pressure is converted to electrical signal
  - generated, and transmitted at another location such as central control station
  - Continuously monitor pressure changes with time (for rapidly changing pressure)

- Types of pressure transducers:
  - Strain gage pressure transducer
  - Linear-variable differential transformer (LVDT) pressure transducer
  - Piezoelectric pressure transducers
  - Quartz resonator pressure transducers

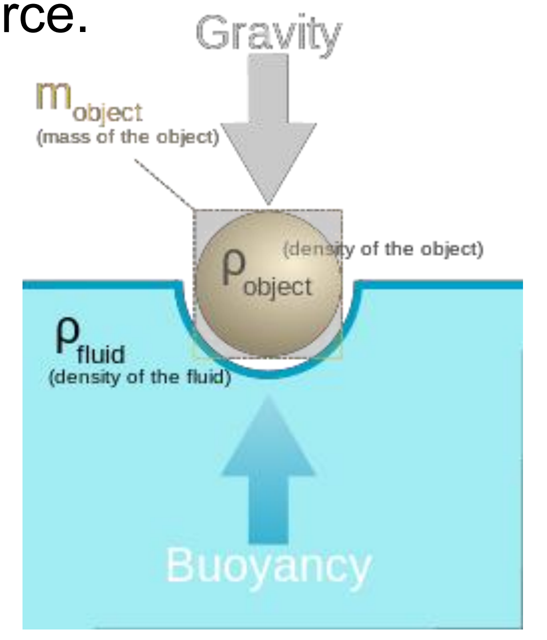
# Buoyancy

## Objectives:

- To derive the equation of buoyant force
- To analyze the case of bodies floating on a fluid
- Use the principle of static equilibrium to solve forces involved in buoyancy problems.

# What is Buoyancy

When an object is submerged or floating in a static fluid the resultant force exerted on it by the fluid is called buoyancy force.

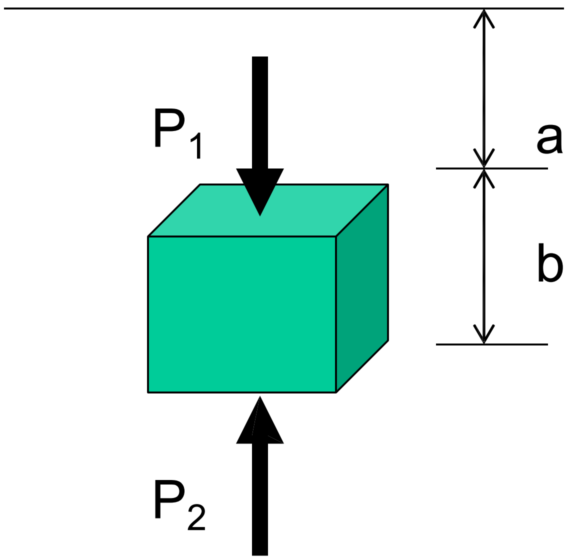



In a column of fluid, **pressure increases with depth** as a result of the weight of the overlying fluid. Thus a column of fluid, or an object submerged in the fluid, experiences greater pressure at the bottom of the column than at the top. This difference in pressure results in a net force that tends to accelerate an object upwards.

The magnitude of buoyancy force is proportional to the difference in the pressure between the top and the bottom of the column, and (as explained by Archimedes' principle) is also equivalent to the weight of the fluid that would otherwise occupy the column, *i.e.* the displaced fluid.

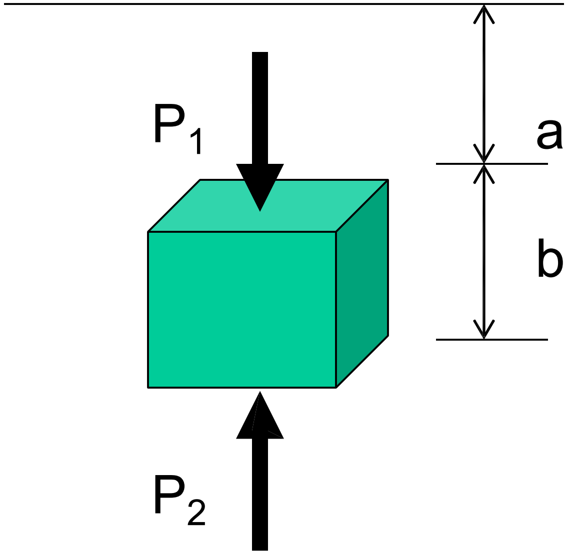
**Archimedes' Principle:** Any Object, wholly or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object

# Buoyant force – Principle



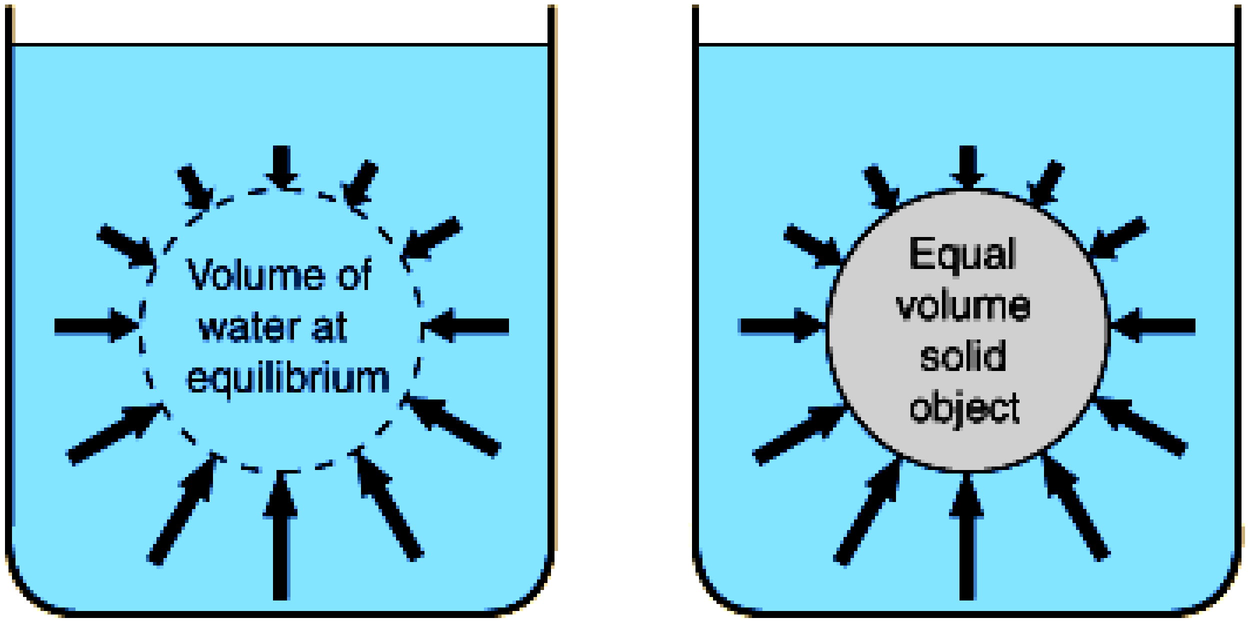

- The buoyant force is caused by the increase of pressure in a fluid with depth.
- Consider a body with a thickness, b is submerged in a liquid of density, $\rho_f$.
- The hydrostatic forces:
  $F_1 = P_1A$ acting downward
  $F_2 = P_2A$ acting upward

# Buoyant force – Principle (Cont'd)




- Buoyant force = the difference between the two forces i.e the net upward force

$$
\begin{aligned}
F_B &= P_2A - P_1A \\
&= \rho_f\, g(a+b)A - \rho_f\, gaA \\
&= \rho_f\, gbA \\
&= \rho_f\, gV_b
\end{aligned}
$$




The **buoyant force** on the solid object is equal to the **weight of the fluid displaced** (**Archimedes' principle**)

For <u>fully submerged</u> or <u>immersed</u> body in a fluid , Archimedes’ principle is restated as:

***The buoyant force on a completely submerged body is equal to the weight of fluid displaced***

$$F_B = W = \rho_f g V_b = \gamma_f V_b$$

**Buoyant force** = Weight of fluid displaced

With respect to the fluid dispersed:

$$F_B = W = \rho_b g V_f = \gamma_b V_f$$

$$F_B = \rho_f g V_{body}$$

$$= \rho_f g \left( \frac{m_{body}}{\rho_{body}} \right)$$

$$= m_{body} g \left( \frac{\rho_f}{\rho_{body}} \right)$$

# Example

- A piece of irregularly shaped metal weighs 300.0 N in air. When the metal is completely submerged in water, it weighs 232.5 N. find the volume of the metal.

Solution:

$$F_B = W = 300 - 232.5 = 67.5\ \text{N}$$

$$F_B = \rho_f g V = (1000)(9.81)\ V = 67.5$$

$$V = 0.00688\ \text{m}^3$$

For floating bodies, Archimedes’ principle is restated as:

*A floating body displaces a volume of fluid whose weight is exactly equal to its own*

The weight of the entire body must be equal to the buoyant force

(Weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body).

- For floating bodies,

$$F_B = W$$

$$\Rightarrow \rho_f g V_{body,sub} = \rho_{avg,\ body} g V_{total}$$

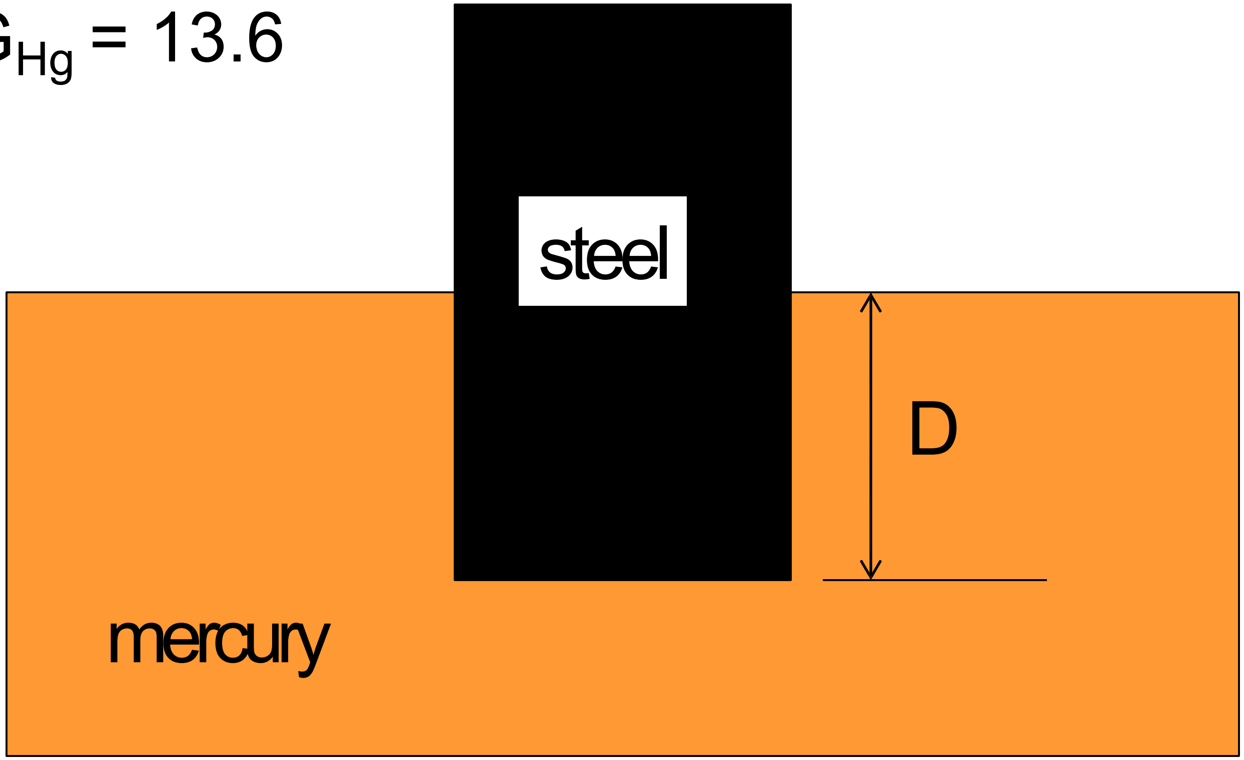
$$\frac{V_{body,sub}}{V_{total}} = \frac{\rho_{avg,\ body}}{\rho_f}$$

# Example

- Determine the submerged depth of a cube of steel 0.30 m on each side floating in mercury.

Given $SG_{steel} = 7.8$

$SG_{Hg} = 13.6$



# Solution

- $F_B = W$

$$V_{body,total} = 0.3 \times 0.3 \times 0.3 = 0.027\ m^3$$

$$\rho_f\, gV_{body,sub} = \rho_{body}\, gV_{body,total}$$

$$(13.6)(1000)(9.81)(0.3 \times 0.3 \times D) = (7.8)(1000)(9.81)(0.027)$$

$$D = 0.172\ m$$

# Example

- A solid block (SG = 0.9) floats such that 75% of its volume is in water and 25% of its volume is in liquid X, which is layered above the water. Determine the density of liquid X.

# Solution

- $F_B = W$

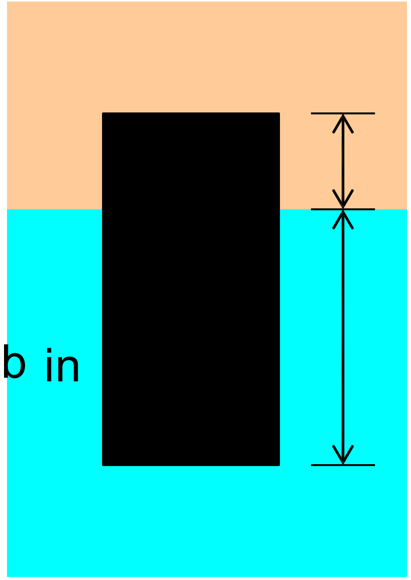
  $\rho_b g V_f = \rho_f g V_b$

  $\rho_b g(V_w + V_X) = \rho_w g V_{b,\text{sub in w}} + \rho_X g V_{b,\text{sub in}}$

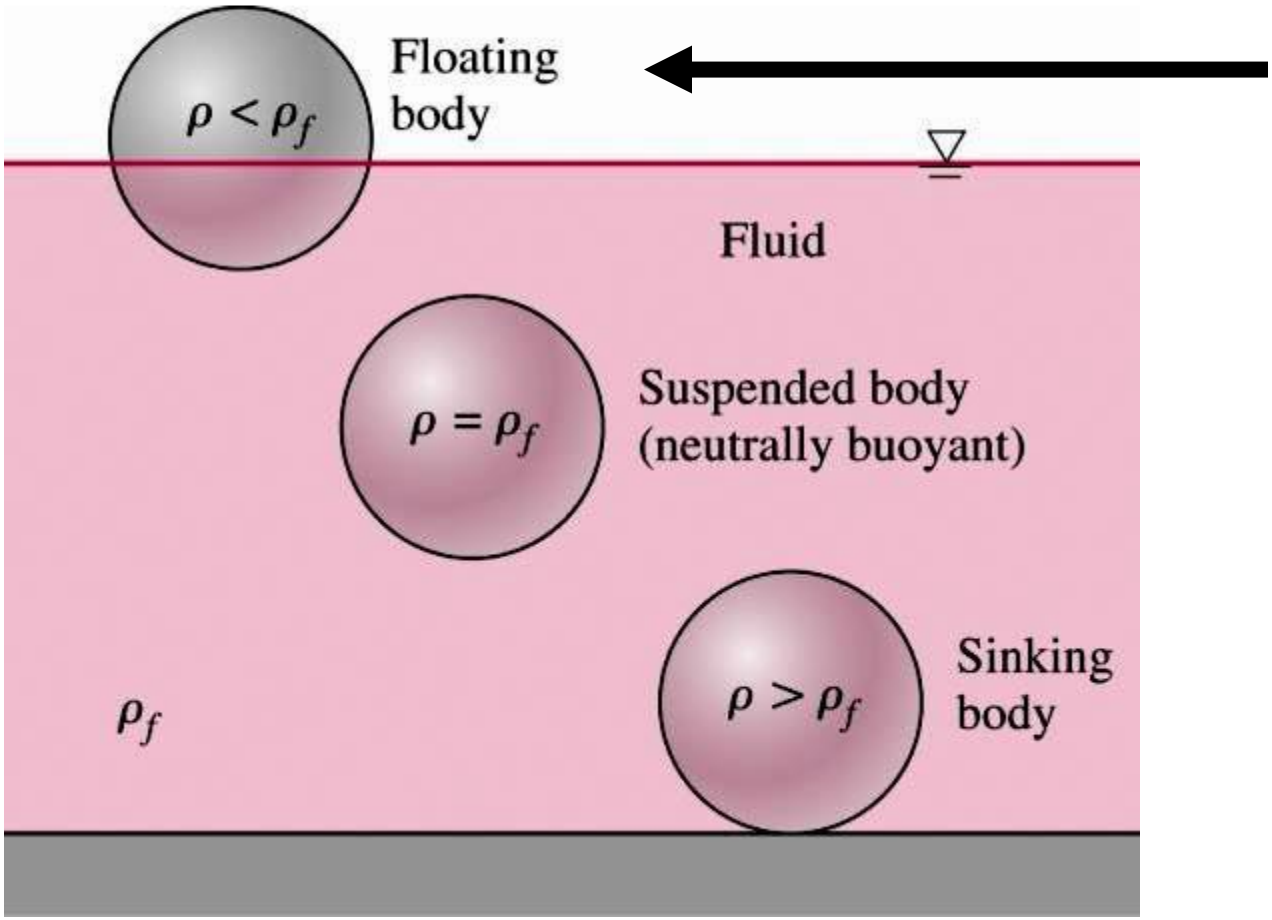
  $\rho_b V_b = \rho_w V_{b,\text{sub in w}} + \rho_X V_{b,\text{sub in X}}$

  $0.9V_b = (1.0)(0.75V_b) + (SG_X)(0.25V_b)$

  $SG_X = 0.60 \rightarrow \rho_X = 600 \text{ kg/m}^3$

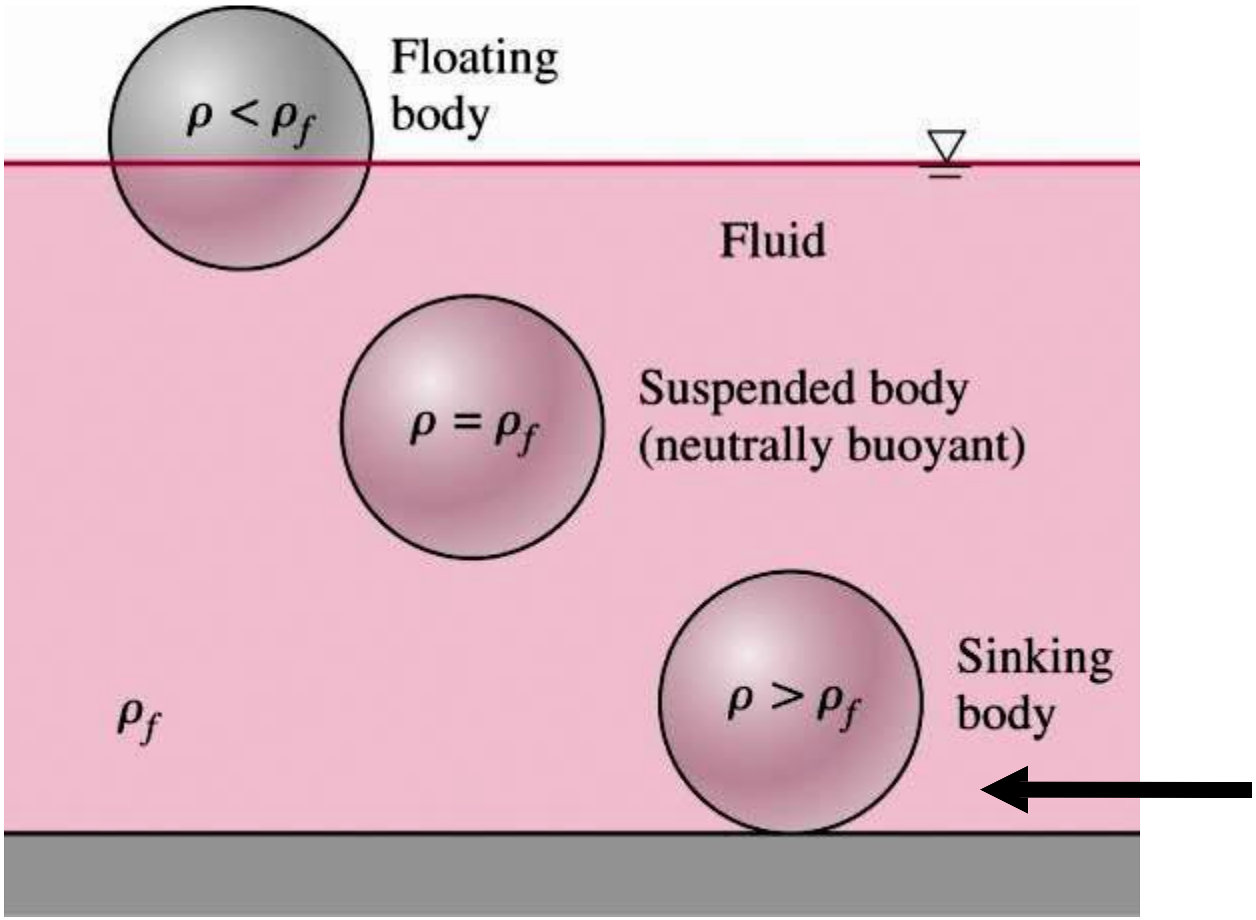


# Sink or Float



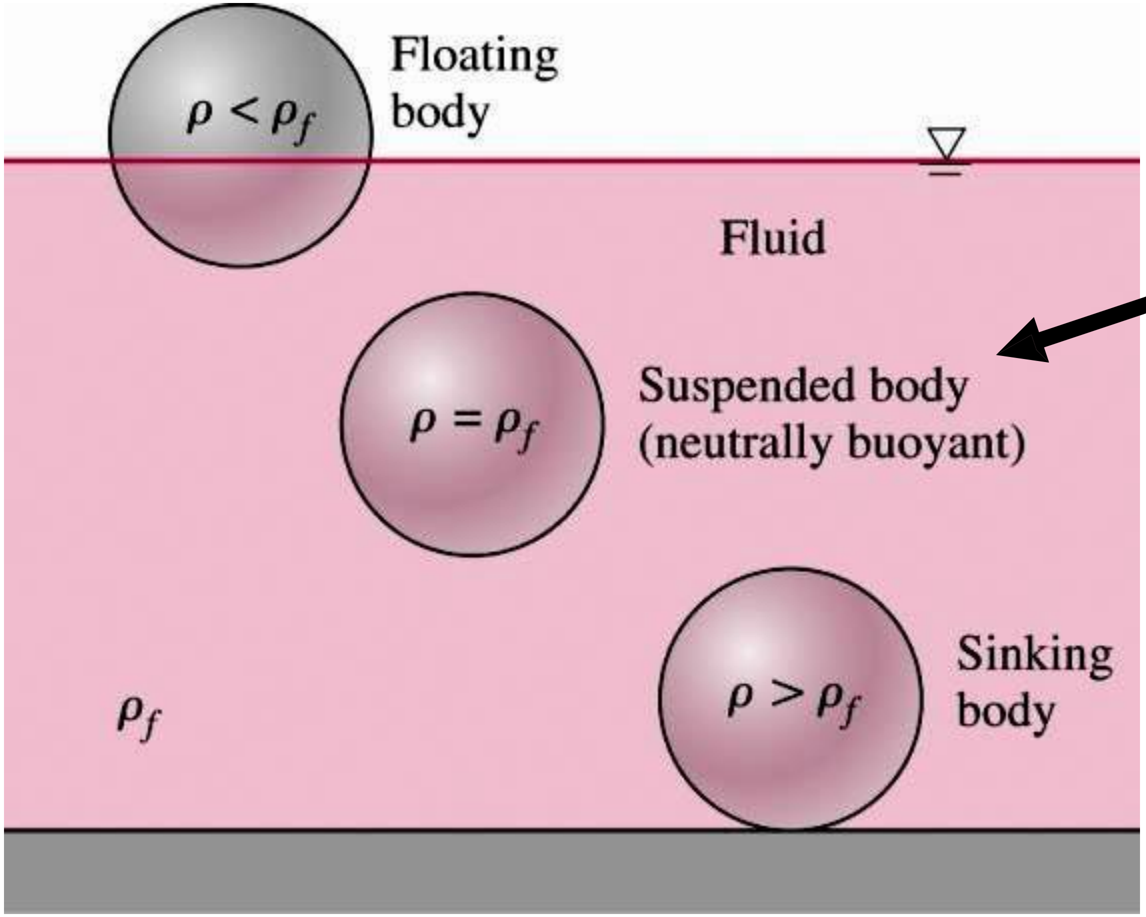

An object with an average specific weight less than that of the fluid tend to float because $W < F_B$

# Sink or Float




An object with an average specific weight greater than that of the fluid tend to sink because $W > F_B$.

# Sink or Float




An object whose average specific weight is equal to that of the fluid is neutrally buoyant.

## What we leant in this chapter (Outcomes)

- ✓ What is fluid statics and hydrostatic equilibrium?
- ✓ Pascal Law
- ✓ How to derive Barometric equation
- ✓ Definition of incompressible and compressible fluids?
- ✓ Pressure depth relationships for fluids
- ✓ Application of fluids statics

  (a) pressure measurement using manometer

  (b) Buoyancy force determination