# INTERNAL FLOW

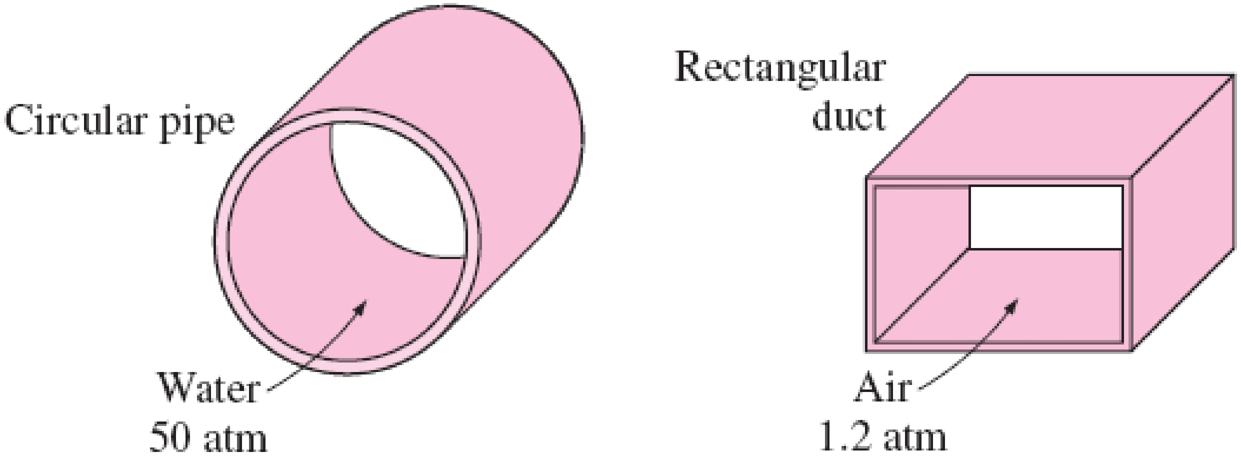
# Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
- Understand various velocity and flow rate measurement techniques and learn their advantages and disadvantages

# 14-1 ■ INTRODUCTION

- Fluid flow is classified as external or internal, depending on whether the fluid is forced to flow over a surface or in a conduit.
- Internal and external flows exhibit very different characteristics. In this chapter we consider internal flow where the conduit is completely filled with the fluid, and the flow is driven primarily by a pressure difference.

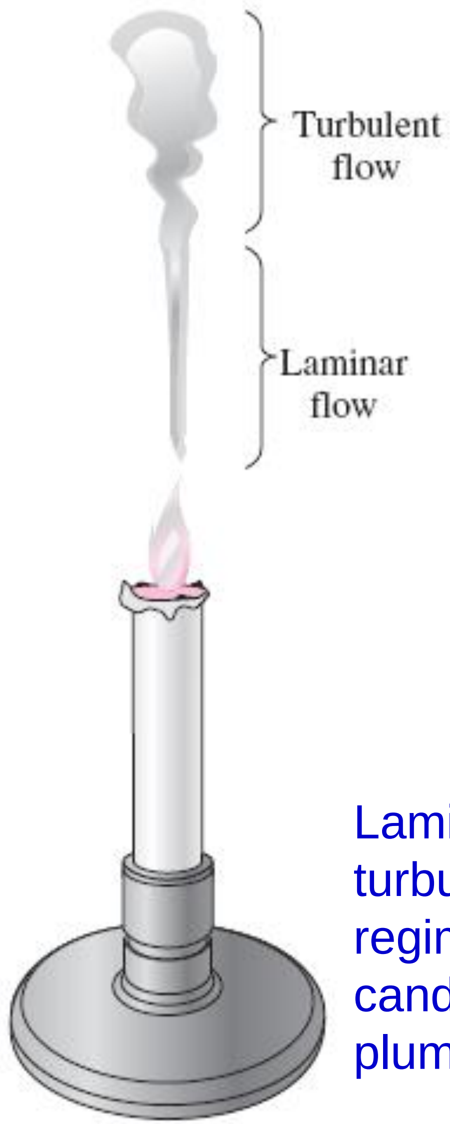
- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.




Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

# 14–2 ■ LAMINAR AND TURBULENT

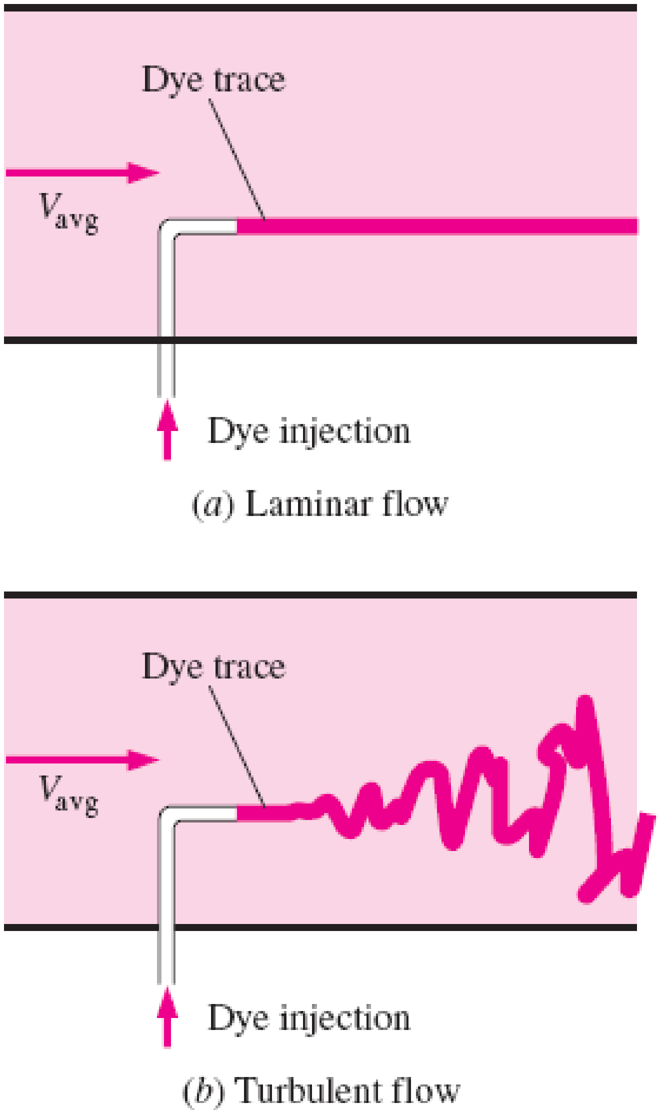
Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.




**Laminar:** Smooth streamlines and highly ordered motion.
**Turbulent:** Velocity fluctuations and highly disordered motion.
**Transition:** The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.

Laminar and turbulent flow regimes of candle smoke plume.




The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

# Reynolds Number

- After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the ratio of inertial forces to viscous forces in the fluid.

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

- where $V_{avg}$ = average flow velocity (m/s), D = characteristic length of the geometry (diameter in this case, in m), and $\nu = \mu/\rho$ = kinematic viscosity of the fluid ($m^2/s$).

# Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$

where $V_{avg}$ = average flow velocity (m/s), D = characteristic length of the geometry (diameter in this case, in m), and n = m/r = kinematic viscosity of the fluid ($m^2$/s).

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid “in line” (**laminar**).

**Critical Reynolds number, $\mathbf{Re_{cr}}$:** The Reynolds number at which the flow becomes turbulent.

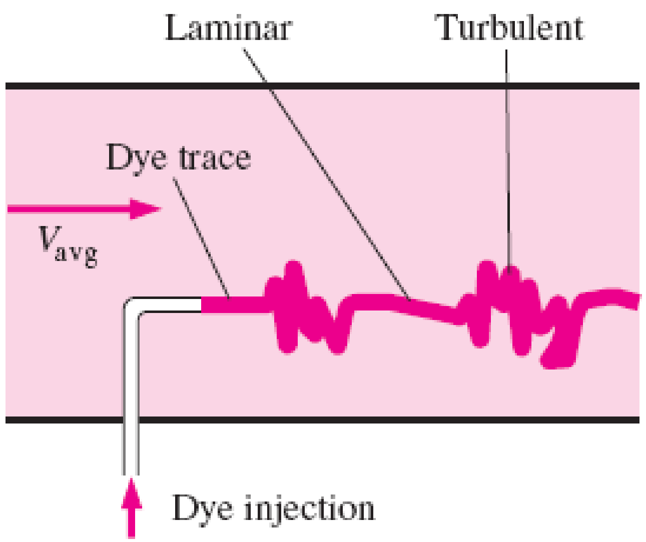
The value of the critical Reynolds number is different for different geometries and flow conditions.

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter**

$$D_h = \frac{4A_c}{p}$$

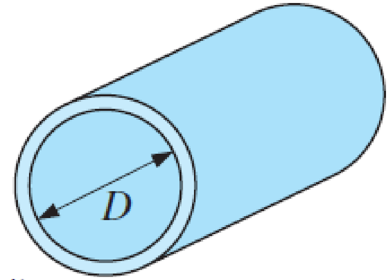
For flow in a circular pipe:

$$\text{Re} \lesssim 2300 \qquad \text{laminar flow}$$
$$2300 \lesssim \text{Re} \lesssim 4000 \qquad \text{transitional flow}$$
$$\text{Re} \gtrsim 4000 \qquad \text{turbulent flow}$$



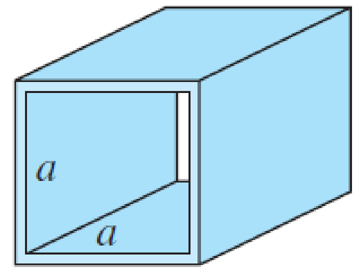

In the transitional flow region of 2300 ≤ Re ≤ 4000, the flow switches between laminar and turbulent randomly.

Circular tube:



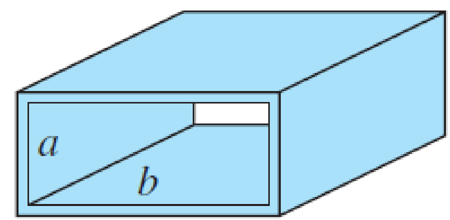

$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:




$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:




$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

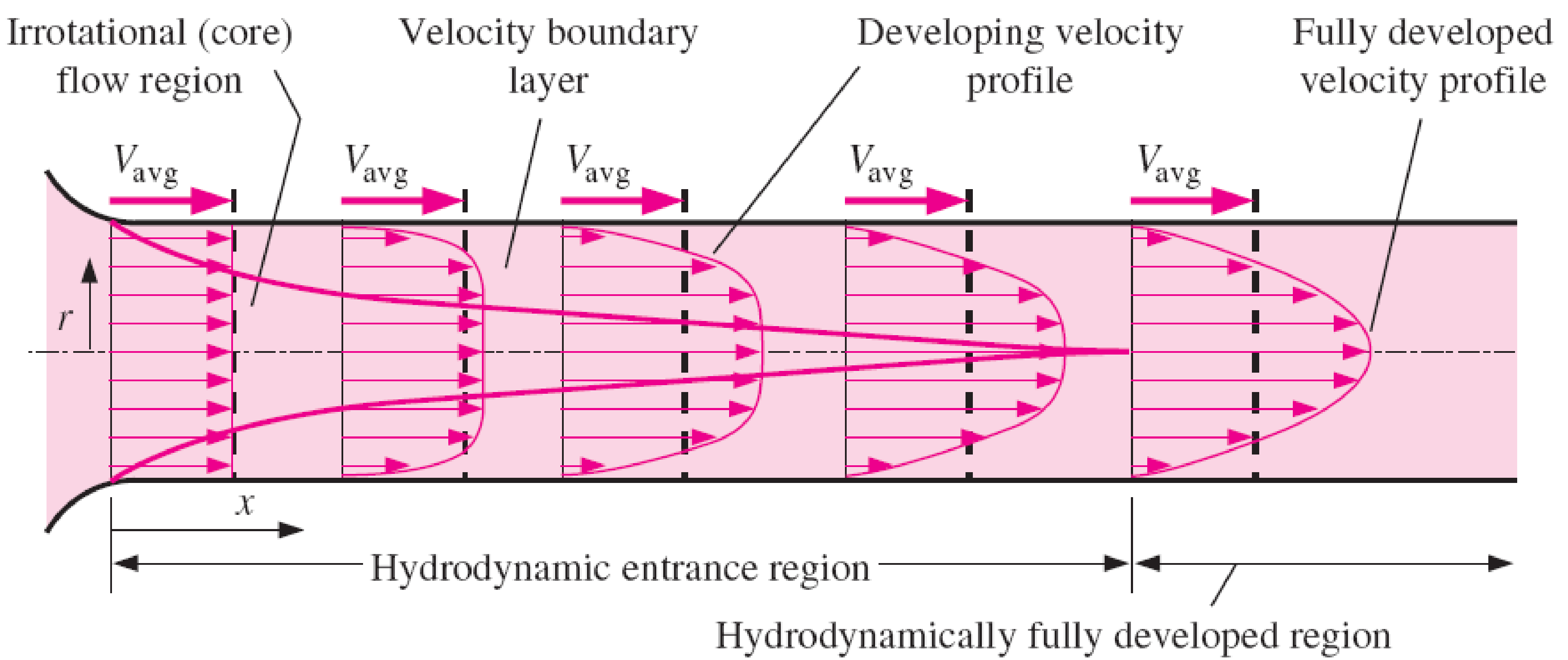
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

# 14–3 ■ THE ENTRANCE REGION

**Velocity boundary layer:** The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

**Boundary layer region:** The viscous effects and the velocity changes are significant.

**Irrotational (core) flow region:** The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



**Hydrodynamic entrance region:** The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

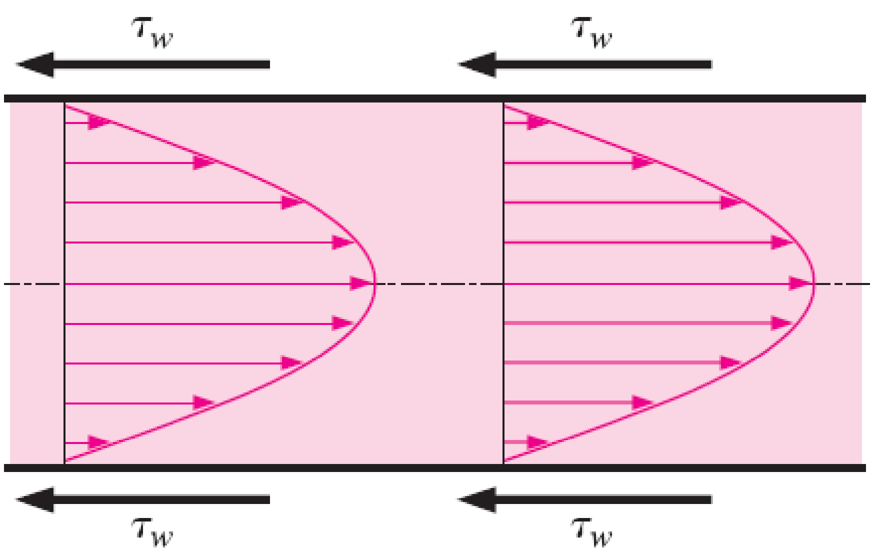
**Hydrodynamic entry length $L_h$:** The length of this region.

**Hydrodynamically developing flow:** Flow in the entrance region. This is the region where the velocity profile develops.

**Hydrodynamically fully developed region:** The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

**Fully developed:** When both the velocity profile and the normalized temperature profile remain unchanged.

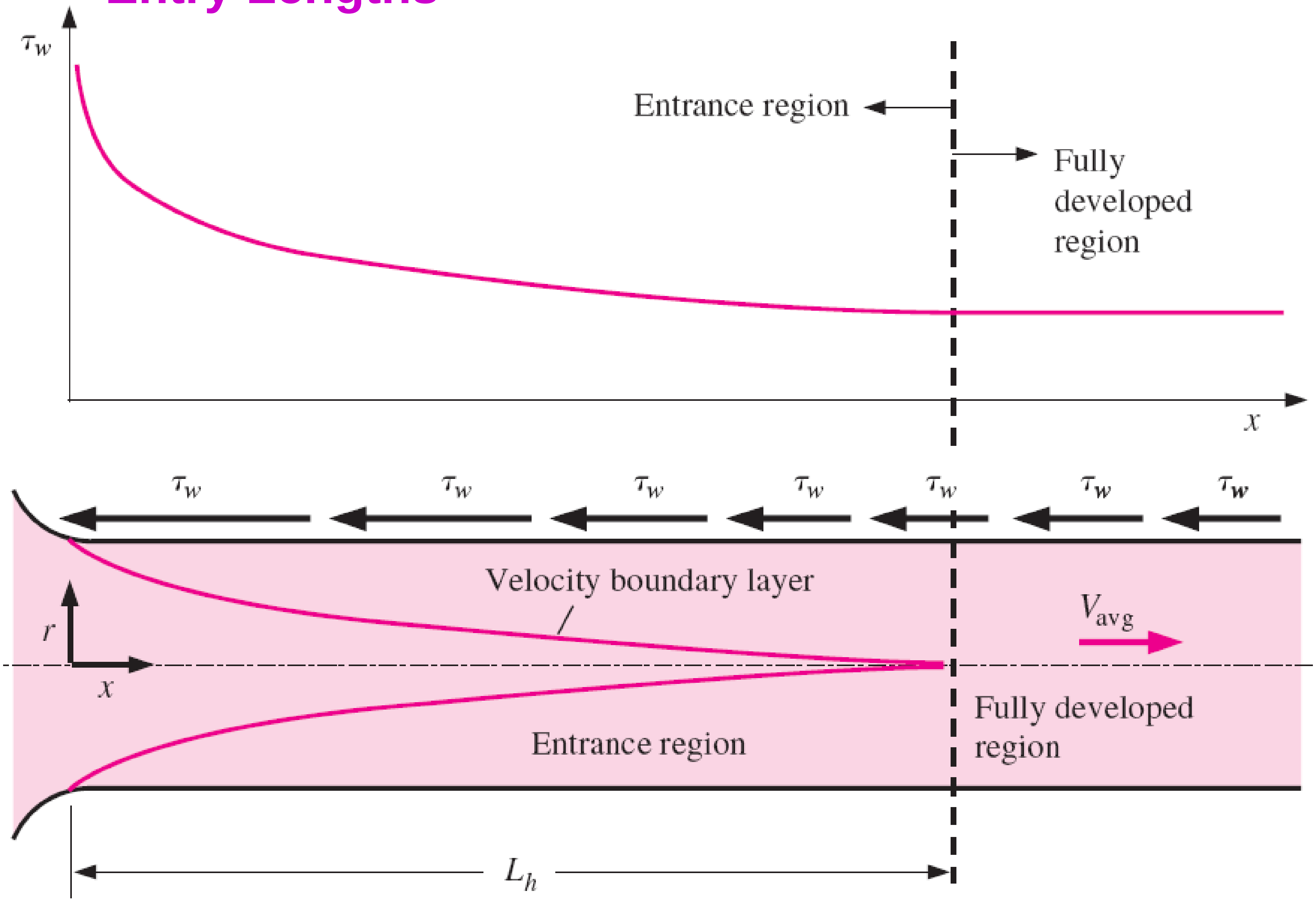
## *Hydrodynamically fully developed*




$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

# Entry Lengths




The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

## Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

$$\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05\text{Re}$$

hydrodynamic entry length for laminar flow

$$\frac{L_{h,\,\text{turbulent}}}{D} = 1.359\text{Re}^{1/4}$$

hydrodynamic entry length for turbulent flow

$$\frac{L_{h,\,\text{turbulent}}}{D} \approx 10$$

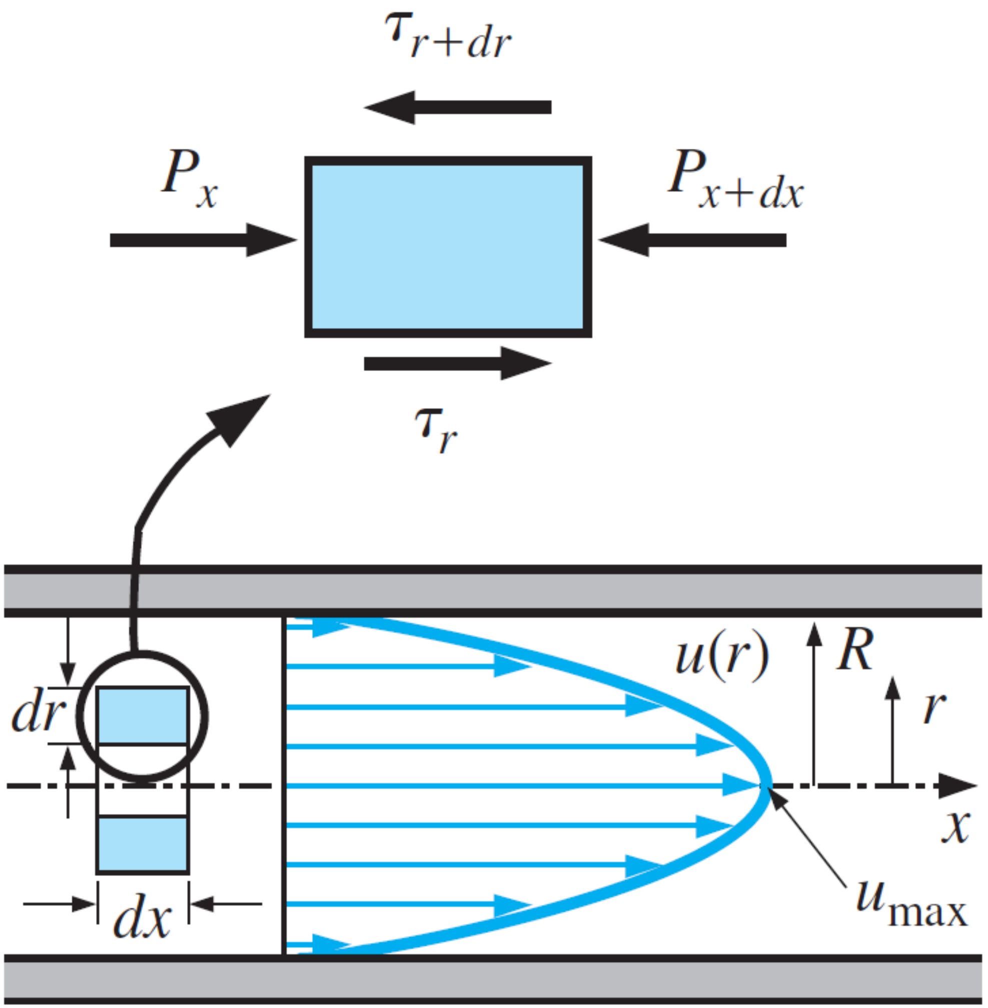
hydrodynamic entry length for turbulent flow, an approximation

# 14–4 ■ LAMINAR FLOW IN PIPES

- Assumptions:
  - steady, laminar, incompressible, constant properties, fully developed flow in a straight circular pipe.
  - There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero.
  - There is no acceleration since the flow is steady and fully developed.
- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction.

# 14–4 ■ LAMINAR FLOW IN PIPES

Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.




$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

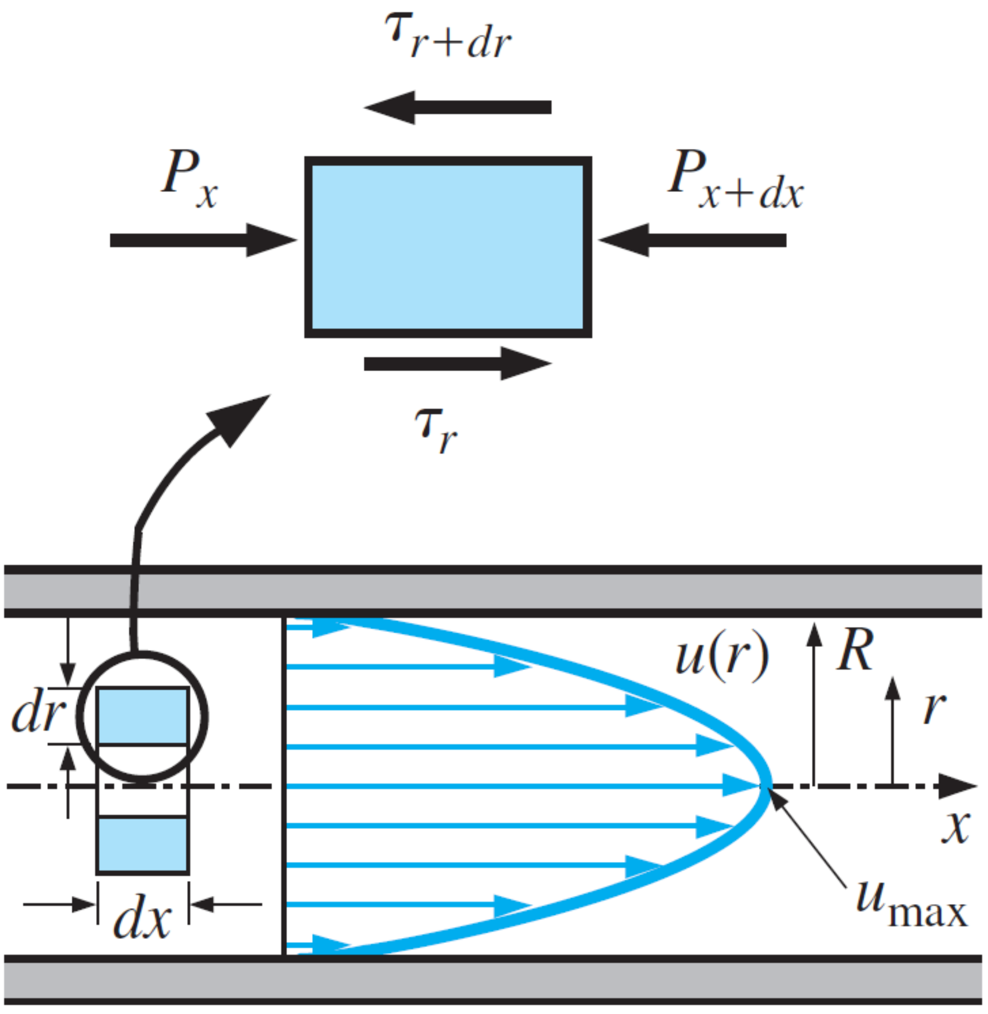
# 14–4 ■ LAMINAR FLOW IN PIPES

the velocity profile in fully developed laminar flow in a pipe is *parabolic* with a maximum at the centerline and a minimum (zero) at the pipe wall.

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$u_{max} = 2V_{avg}$$

**Therefore, *the average velocity in fully developed laminar pipe flow is one half of the maximum velocity.***



# Pressure Drop

**A quantity of interest in the analysis of pipe flow is the pressure drop ΔP since it is directly related to the power requirements of the fan or pump to maintain flow.**

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible losses, and it is called **pressure loss $\Delta P_L$**.

# Pressure Drop

Pressure loss for all types of fully developed internal flows (laminar / turbulent flows, circular/ non-circular pipes, smooth / rough surfaces, and horizontal or inclined pipes)

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{\text{avg}}^2}{2}$$

Dynamic pressure

$$\frac{\rho V_{\text{avg}}^2}{2}$$

Darcy friction factor

$$f = \frac{8\tau_w}{\rho V_{\text{avg}}^2}$$

Circular pipe, laminar

$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}}$$

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

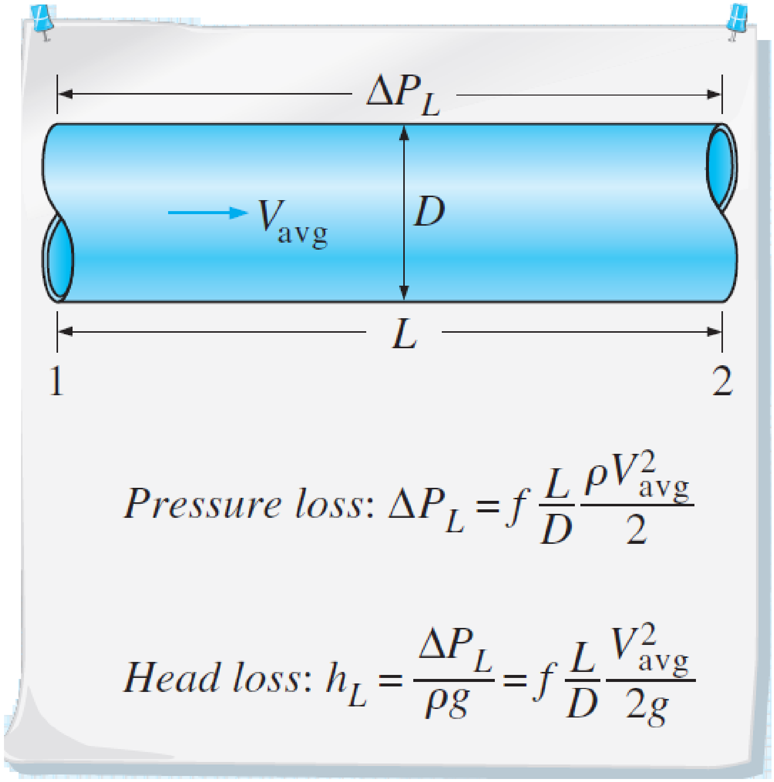
# Head Loss and Pumping Power

The **head loss** represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

Head loss (laminar/turbulent, circular/non-circular pipes)

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.




The required pumping power to overcome the pressure loss.

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V} \rho g h_L = \dot{m} g h_L$$

# Laminar Flow (Horizontal pipe)

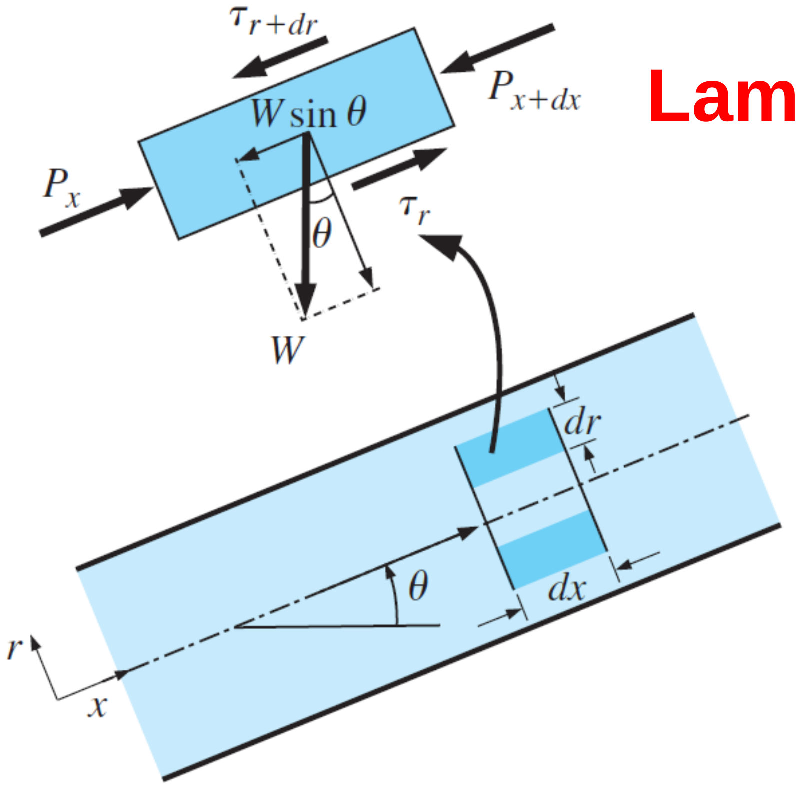
$$V_{\text{avg}} = \frac{\Delta P\, D^2}{32\mu L}$$

$$\dot{V} = V_{\text{avg}} A_c = \frac{\Delta P\, \pi D^4}{128\mu L}$$

**Poiseuille's law**

For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.

# Effect of Gravity on Velocity and Flow Rate in Laminar Flow




Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

## Laminar Flow (Inclined pipe)

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

## Laminar Flow in Circular Pipes

(Fully developed flow with no pump or turbine in the flow section, and $\Delta P = P_1 - P_2$)

$$\text{Horizontal pipe: } \dot{V} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

$$\text{Inclined pipe: } \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing $\Delta P$ with $\Delta P - \rho g L \sin\theta$.

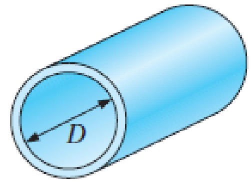
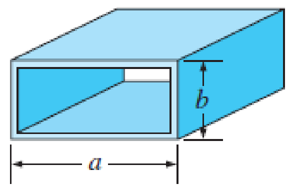
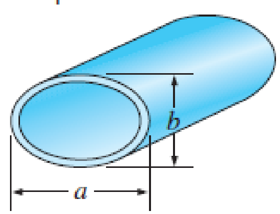
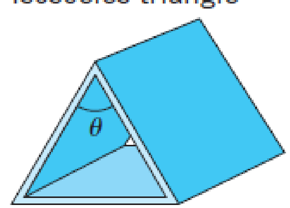
# Laminar Flow in Noncircular Pipes

The friction factor *f* relations are given in Table 14–1 for fully developed laminar flow in pipes of various cross sections.

The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where $A_c$ is the cross-sectional area of the pipe and p is its wetted perimeter

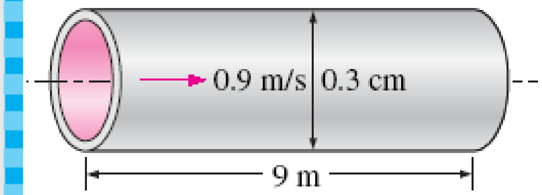
**TABLE 14–1**

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{avg} D_h/\nu$)

| Tube Geometry | $a/b$ or $\theta°$ | Friction Factor $f$ |
|---|---|---|
| Circle   | — | 64.00/Re |
| Rectangle   | $a/b$ | |
| | 1 | 56.92/Re |
| | 2 | 62.20/Re |
| | 3 | 68.36/Re |
| | 4 | 72.92/Re |
| | 6 | 78.80/Re |
| | 8 | 82.32/Re |
| | ∞ | 96.00/Re |
| Ellipse   | $a/b$ | |
| | 1 | 64.00/Re |
| | 2 | 67.28/Re |
| | 4 | 72.96/Re |
| | 8 | 76.60/Re |
| | 16 | 78.16/Re |
| Isosceles triangle   | $\theta$ | |
| | 10° | 50.80/Re |
| | 30° | 52.28/Re |
| | 60° | 53.32/Re |
| | 90° | 52.60/Re |
| | 120° | 50.96/Re |

**EXAMPLE 14–2 Pressure Drop and Head Loss in a Pipe**

Water at 5°C ($\rho$ = 1000 kg/m$^3$ and $\mu$ = 1.519 × 10$^{-3}$ kg/m·s) is flowing steadily through a 0.3-cm diameter 9-m-long horizontal pipe at an average velocity of 0.9 m/s (Fig. 14–18). Determine (*a*) the head loss, (*b*) the pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.




**Solution** The average flow velocity in a pipe is given. The head loss, the pressure drop, and the pumping power are to be determined.

***Assumptions*** **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

***Properties*** The density and dynamic viscosity of water are given to be $\rho$ = 1000 kg/m$^3$ and $\mu$ = 1.519 × 10$^{-3}$ kg/m·s, respectively.

***Analysis*** (*a*) First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(1000 \text{ kg/m}^3)(0.9 \text{ m/s})(0.003 \text{ m})}{1.519 \times 10^{-3} \text{ kg/m·s}} = 1777$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{1777} = 0.0360$$

$$h_L = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g} = 0.0360 \frac{9 \text{ m}}{0.003 \text{ m}} \frac{(0.9 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{4.46 \text{ m}}$$

(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0360\frac{9\text{ m}}{0.003\text{ m}}\frac{(1000\text{ kg/m}^3)(0.9\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right)$$

$$= \mathbf{43{,}740\ N/m^2 = 43.7\ kPa}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi D^2/4) = (0.9\text{ m/s})[\pi(0.003\text{ m})^2/4] = 6.36\times 10^{-6}\text{ m}^3/\text{s}$$

$$\dot{W}_{pump} = \dot{V}\,\Delta P = (6.36\times 10^{-6}\text{ m}^3/\text{s})(43{,}740\text{ N/m}^2)\left(\frac{1\text{ W}}{1\text{ N·m/s}}\right) = \mathbf{0.28\ W}$$

Therefore, power input in the amount of 0.28 W is needed to overcome the frictional losses in the flow due to viscosity.

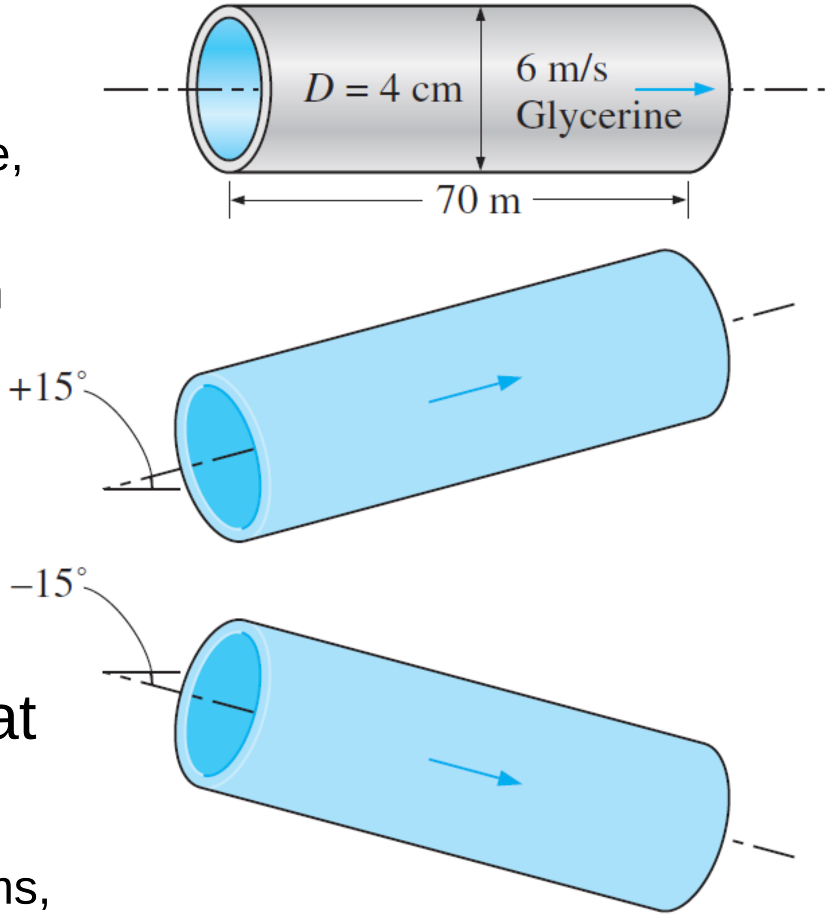
***Discussion*** The pressure rise provided by a pump is often listed by a pump manufacturer in units of head. Thus, the pump in this flow needs to provide 4.46 m of water head in order to overcome the irreversible head loss.

# EXAMPLE 14–1 Laminar Flow in Horizontal and Inclined Pipes

Consider the fully developed flow of glycerin at 40$^o$C, through a 70-m-long, 4-cm-diameter, horizontal, circular pipe. If the flow velocity at the centerline is measured to be 6 m/s, determine the velocity profile and the pressure difference across this 70-m-long section of the pipe, and the useful pumping power required to maintain this flow. For the same useful pumping power input, determine the percent increase of the flow rate if the pipe is inclined 15 downward and the percent decrease if it is inclined 15 upward. The pump is located outside this pipe section.

# Solution

- **Assumptions**
  - ✓ **1** The flow is steady, incompressible, and fully developed.
  - ✓ **2** There are no pumps or turbines in the flow section.
  - ✓ **3** There are no valves, elbows, or other devices that may cause local losses.
- **Properties** The density and dynamic viscosity of glycerin at 40ºC are
  - ✓ $\rho$ = 1252 kg/m$^3$ and $\mu$ = 0.3073 kg/ms, respectively.



$$V = V_{avg} = \frac{u_{max}}{2} = \frac{6\ m/s}{2} = 3\ m/s$$

$$\dot{V} = V_{avg}\ A_c = V(\pi D^2/4) = (3\ \text{m/s})[\pi(0.04\ \text{m})^2/4]$$

$$= 3.77 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1252\ \text{kg/m}^3)(3\ \text{m/s})(0.04\ \text{m})}{0.3073\ \text{kg/m·s}} = 488.9$$

which is less than 2300. Therefore, the flow is indeed laminar.

- Then the friction factor and the head loss become

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{488.9} = 0.1309$$

$$h_L = f\frac{LV^2}{D\,2g} = 0.1309\,\frac{(70\text{ m})}{(0.04\text{ m})}\,\frac{(3\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 105.1\text{m}$$

The energy balance for steady, incompressible one-dimensional flow is given by Eq. 8–28 as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

For fully developed flow in a constant diameter pipe with no pumps or turbines, it reduces to

$$\Delta P = P_1 - P_2 = \rho g(z_2 - z_1 + h_L)$$

Then the pressure difference and the required useful pumping power for the horizontal case become

$$\Delta P = \rho g(z_2 - z_1 + h_L)$$

$$= (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0 + 105.1 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ kg/m} \cdot \text{s}^2}\right)$$

$$= \mathbf{1291\ kPa}$$

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (3.77 \times 10^3 \text{ m}^3\text{/s})(1291 \text{ kPa})\left(\frac{1 \text{ kW}}{\text{kPa} \cdot \text{m}^3\text{/s}}\right) = \mathbf{4.87\ kW}$$

The elevation difference and the pressure difference for a pipe inclined upwards 15° is

$$\Delta z = z_2 - z_1 = L\sin 15° = (70 \text{ m})\sin 15° = 18.1 \text{ m}$$

$$\Delta P_{\text{upward}} = (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(18.1 \text{ m} + 105.1 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ kg/m} \cdot \text{s}^2}\right)$$

$$= 1366 \text{ kPa}$$

Then the flow rate through the upward inclined pipe becomes

$$\dot{V}_{upward} = \frac{\dot{W}_{pump, u}}{\Delta P_{upward}} = \frac{4.87 \text{ kW}}{1366 \text{ kPa}}\left(\frac{1 \text{ kPa} \cdot \text{m}^3/\text{s}}{1 \text{ kW}}\right) = 3.57 \times 10^{-3} \text{ m}^3/\text{s}$$

which is a decrease of **5.6 percent** in flow rate. It can be shown similarly that when the pipe is inclined 15° downward from the horizontal, the flow rate will increase by **5.6 percent**.

***Discussion*** Note that the flow is driven by the combined effect of pumping power and gravity. As expected, gravity opposes uphill flow, enhances downhill flow, and has no effect on horizontal flow. Downhill flow can occur even in the absence of a pressure difference applied by a pump. For the case of $P_1 = P_2$ (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant, and the fluid would flow through the pipe under the influence of gravity at a rate that depends on the angle of inclination, reaching its maximum value when the pipe is vertical. When solving pipe flow problems, it is always a good idea to calculate the Reynolds number to verify the flow regime—laminar or turbulent.

# 14–5 ■ TURBULENT FLOW IN PIPES

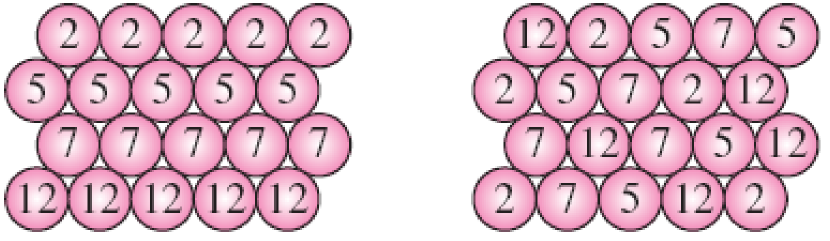
Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients

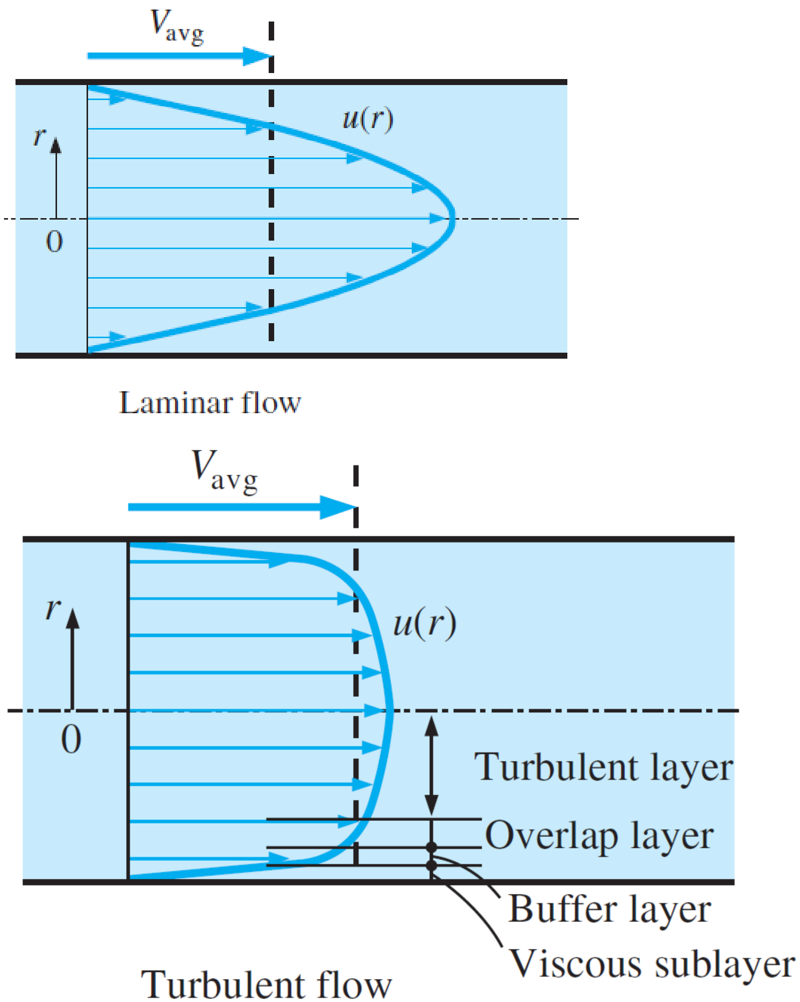



(*a*) Before turbulence

(*b*) After turbulence

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

# Turbulent Velocity Profile




**Viscous sublayer**: the velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

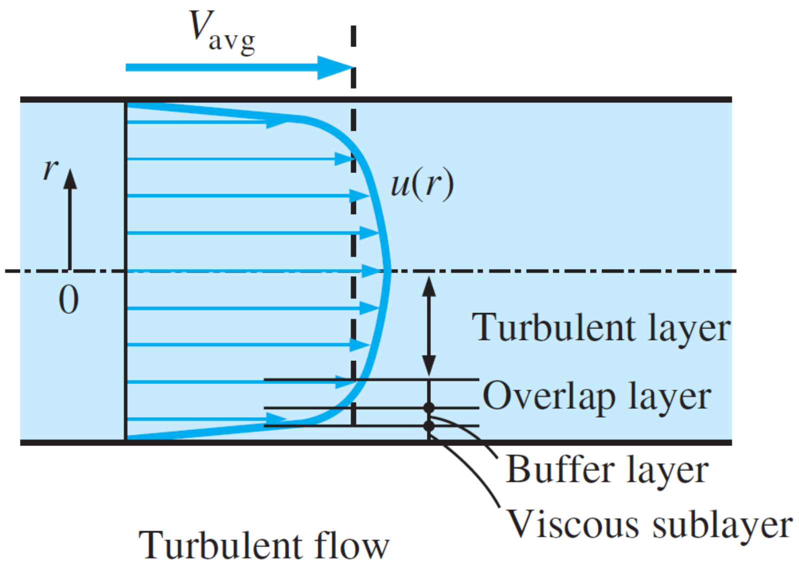
**Buffer layer:** Next to the viscous sublayer, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

**Overlap** (or **transition**) **layer:** Above the buffer layer, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant.

**Outer** (or **turbulent**) **layer:** In the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow.

# Turbulent Velocity Profile




- Flow characteristics are quite different in different regions, and thus it is difficult to come up with an analytic relation for the velocity profile for the entire flow as we did for laminar flow.

- Several equations have been generated in this manner to approximate the velocity profile shape in fully developed turbulent pipe flow. Discussion of these equations is beyond the scope of the present text.

# The Moody Chart and the Colebrook Equation

- The friction factor $(f)$ in fully developed turbulent pipe flow depends on the **Reynolds number** and the **relative roughness** $\varepsilon/D$, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.
- The functional form of this dependence cannot be obtained from a theoretical analysis
- The friction factor was calculated from measurements of the flow rate and the pressure drop.

Colebrook equation (for smooth and rough pipes)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \quad \text{(turbulent flow)}$$

Explicit Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

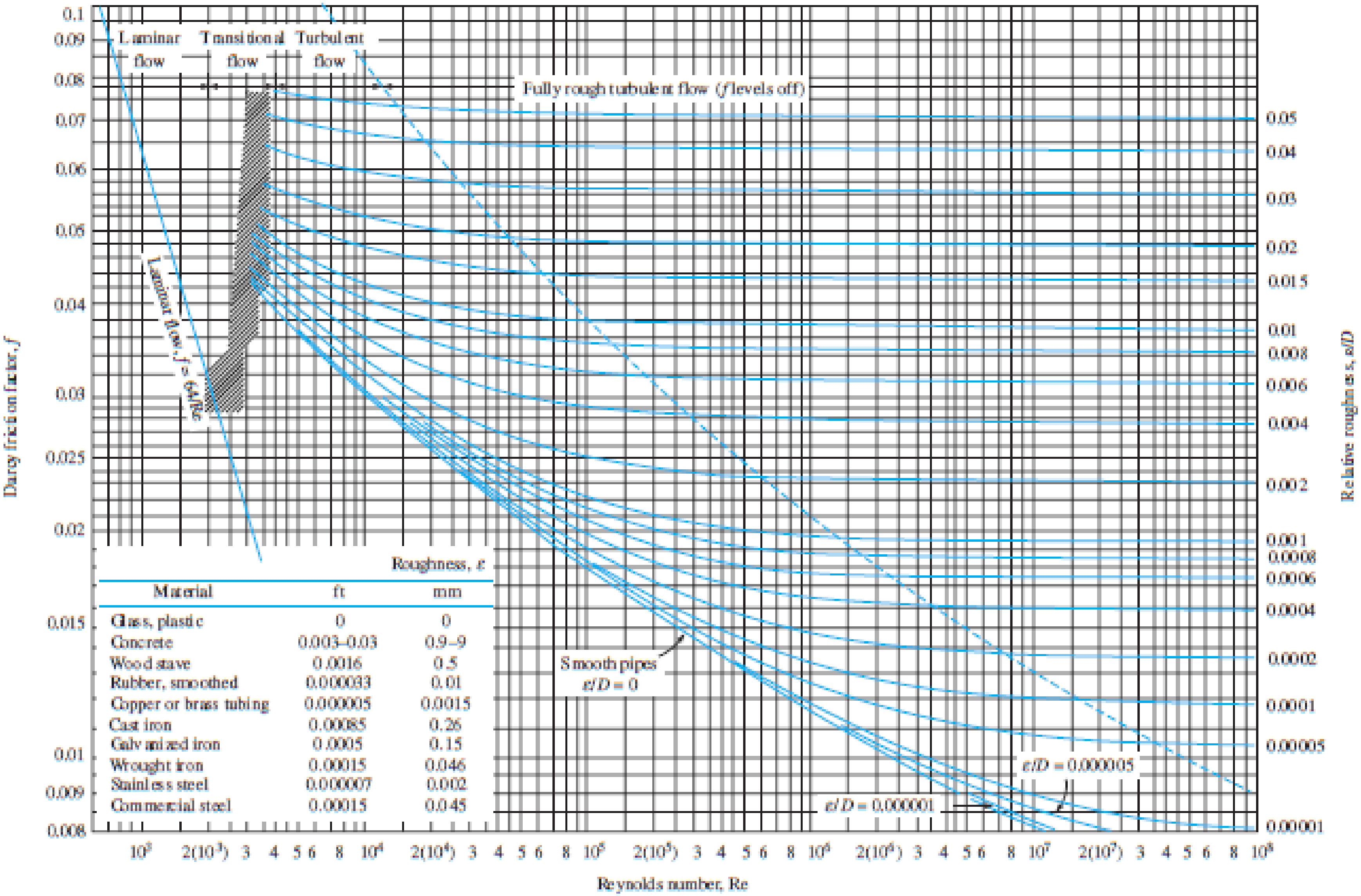
The friction factor for a smooth pipe and Re = $10^6$ from Colebrook eqn.

| Relative Roughness, $\varepsilon/D$ | Friction Factor, $f$ |
|---|---|
| 0.0* | 0.0119 |
| 0.00001 | 0.0119 |
| 0.0001 | 0.0134 |
| 0.0005 | 0.0172 |
| 0.001 | 0.0199 |
| 0.005 | 0.0305 |
| 0.01 | 0.0380 |
| 0.05 | 0.0716 |

* Smooth surface. All values are for Re = $10^6$ and are calculated from the Colebrook equation.

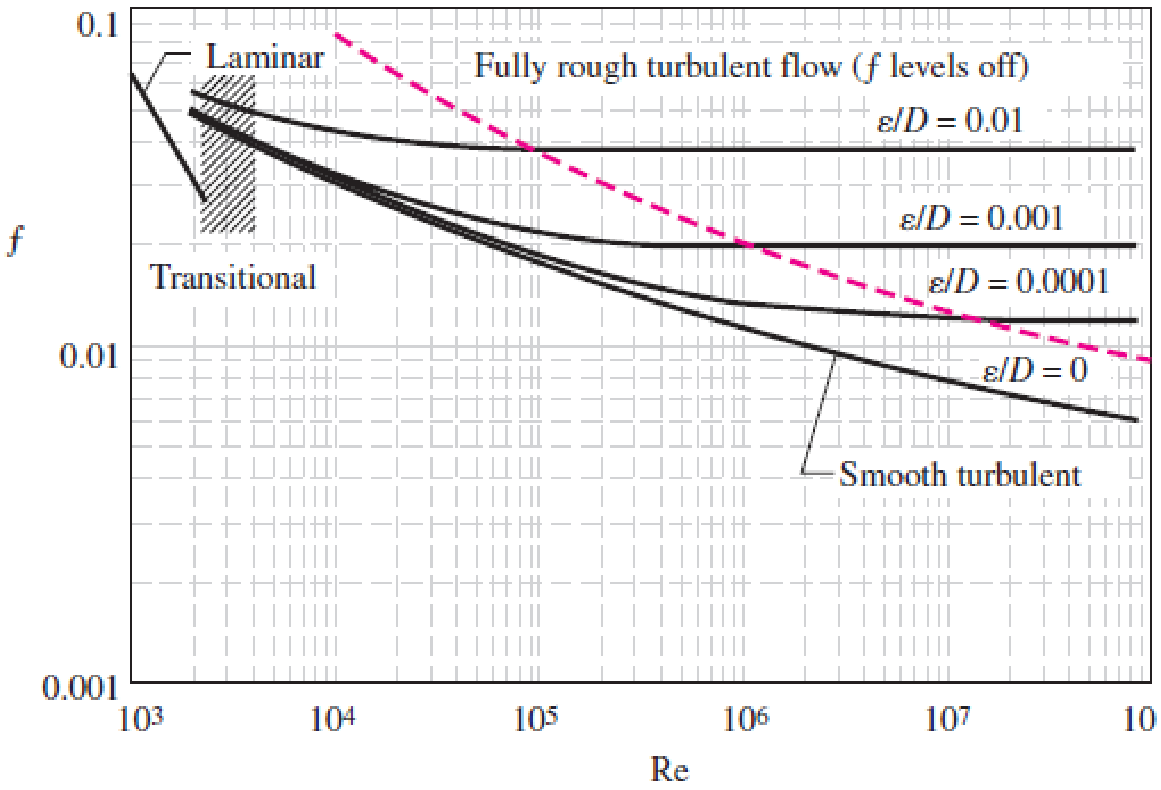
Equivalent roughness values for new commercial pipes*

| | Roughness, $\varepsilon$ | |
|---|---|---|
| Material | ft | mm |
| Glass, plastic | 0 (smooth) | |
| Concrete | 0.003–0.03 | 0.9–9 |
| Wood stave | 0.0016 | 0.5 |
| Rubber, smoothed | 0.000033 | 0.01 |
| Copper or brass tubing | 0.000005 | 0.0015 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Wrought iron | 0.00015 | 0.046 |
| Stainless steel | 0.000007 | 0.002 |
| Commercial steel | 0.00015 | 0.045 |




| Material | Roughness, ε ft | Roughness, ε mm |
|---|---|---|
| Glass, plastic | 0 | 0 |
| Concrete | 0.003–0.03 | 0.9–9 |
| Wood stave | 0.0016 | 0.5 |
| Rubber, smoothed | 0.000033 | 0.01 |
| Copper or brass tubing | 0.000005 | 0.0015 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Wrought iron | 0.00015 | 0.046 |
| Stainless steel | 0.000007 | 0.002 |
| Commercial steel | 0.00015 | 0.045 |

In 1944, Lewis F. Moody (1880–1953) produced the now famous **Moody chart**




In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter.

At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–27 for a full-page moody chart.

Standard sizes for Schedule 40 steel pipes

| Nominal Size, in | Actual Inside Diameter, in |
|---|---|
| $\frac{1}{8}$ | 0.269 |
| $\frac{1}{4}$ | 0.364 |
| $\frac{3}{8}$ | 0.493 |
| $\frac{1}{2}$ | 0.622 |
| $\frac{3}{4}$ | 0.824 |
| 1 | 1.049 |
| $1\frac{1}{2}$ | 1.610 |
| 2 | 2.067 |
| $2\frac{1}{2}$ | 2.469 |
| 3 | 3.068 |
| 5 | 5.047 |
| 10 | 10.02 |

# Types of Fluid Flow Problems

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

| Problem type | Given | Find |
|---|---|---|
| 1 | $L, D, \dot{V}$ | $\Delta P$ (or $h_L$) |
| 2 | $L, D, \Delta P$ | $\dot{V}$ |
| 3 | $L, \Delta P, \dot{V}$ | $D$ |

The three types of problems encountered in pipe flow.

$$h_L = 1.07\frac{\dot{V}^2L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3\times 10^{8} \end{matrix}$$

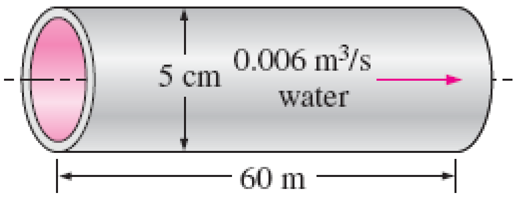
$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2L}{gD^3h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3\times 10^{8} \end{matrix}$$

To avoid tedious iterations in head loss, flow rate, and diameter calculations, these explicit relations that are accurate to within 2 percent of the Moody chart may be used.

## *EXAMPLE 8–3* **Determining the Head Loss in a Water Pipe**

Water at 15°C ($\rho$ = 999 kg/m$^3$ and $\mu$ = 1.138 × 10$^{-3}$ kg/m · s) is flowing steadily in a 5-cm-diameter horizontal pipe made of stainless steel at a rate of 0.006 m$^3$/s (Fig. 8–30). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 60-m-long section of the pipe.




**SOLUTION** The flow rate through a specified water pipe is given. The pressure drop, the head loss, and the pumping power requirements are to be determined.

***Assumptions*** **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

***Properties*** The density and dynamic viscosity of water are given to be $\rho$ = 999 kg/m$^3$ and $\mu$ = 1.138 × 10$^{-3}$ kg/m · s, respectively.

***Analysis*** We recognize this as a problem of the first type, since flow rate, pipe length, and pipe diameter are known. First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 134{,}300$$

Since Re is greater than 4000, the flow is turbulent. The relative roughness of the pipe is estimated using Table 8–2

$$\varepsilon/D = \frac{0.002\ \text{mm}}{50\ \text{mm}} = 0.000040$$

The friction factor corresponding to this relative roughness and Reynolds number is determined from the Moody chart. To avoid any reading error, we determine $f$ from the Colebrook equation on which the Moody chart is based:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.000040}{3.7} + \frac{2.51}{134{,}300\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0172$. Then the pressure drop (which is equivalent to pressure loss in this case), head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0172\frac{60\text{ m}}{0.05\text{ m}}\frac{(999\text{ kg/m}^3)(3.06\text{ m/s})}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{96{,}540\ N/m^2 = 96.5\ kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0172\frac{60\text{ m}}{0.05\text{ m}}\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{9.85\ m}$$

$$\dot{W}_{\text{pump}} = \dot{V}\,\Delta P = (0.006\text{ m}^3/\text{s})(96{,}540\text{ N/m}^2)\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{579\ W}$$

Therefore, power input in the amount of 579 W is needed to overcome the frictional losses in the pipe.

***Discussion*** It is common practice to write our final answers to three significant digits, even though we know that the results are accurate to at most two significant digits because of inherent inaccuracies in the Colebrook equation, as discussed previously. The friction factor could also be determined easily from the explicit Haaland relation (Eq. 8–51). It would give $f = 0.0170$, which is sufficiently close to 0.0172. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0169, which indicates that this stainless-steel pipe can be approximated as smooth with negligible error.

# 14–6 ■ MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, expansions, and contractions in addition to the pipes.

These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the **major losses**) and are called **minor losses**.

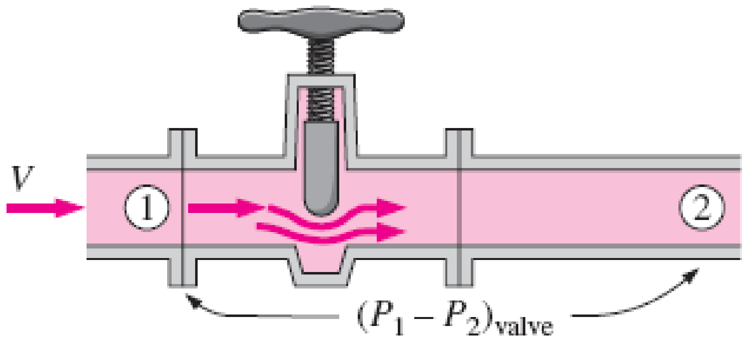
Minor losses are usually expressed in terms of the **loss coefficient $K_L$**.

$$K_L = \frac{h_L}{V^2/(2g)}$$

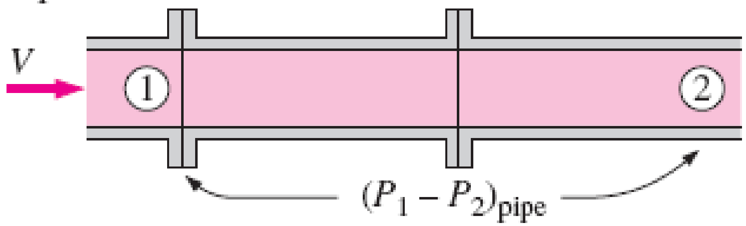
$$h_L = \Delta P_L / \rho g$$

Head loss due to component

Pipe section with valve:




Pipe section without valve:




$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$

For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

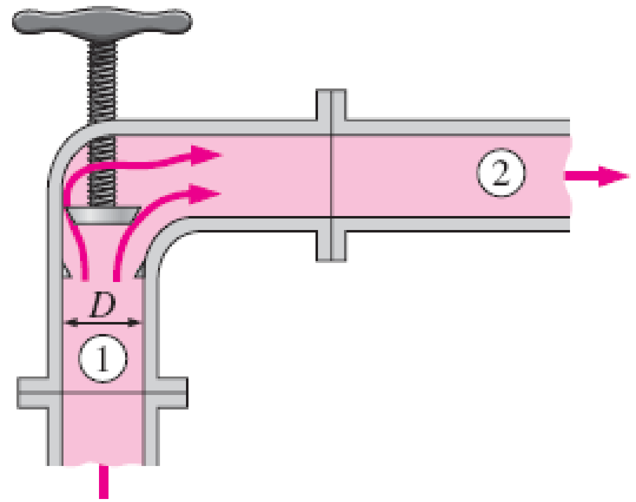
$$K_L = \Delta P_L / (\rho V^2/2).$$

When the loss coefficient for a component is available, the head loss for that component is

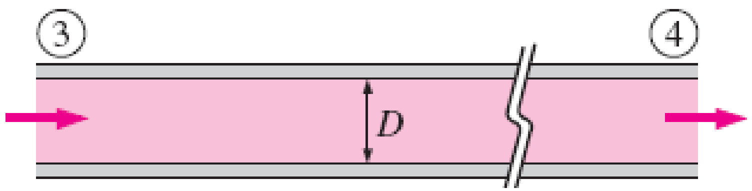
$$h_L = K_L \frac{V^2}{2g}$$ *Minor loss*

Minor losses are also expressed in terms of the **equivalent length $L_{equiv}$**.

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{equiv}}{D} \frac{V^2}{2g} \rightarrow L_{equiv} = \frac{D}{f} K_L$$




$$\Delta P = P_1 - P_2 = P_3 - P_4$$




The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

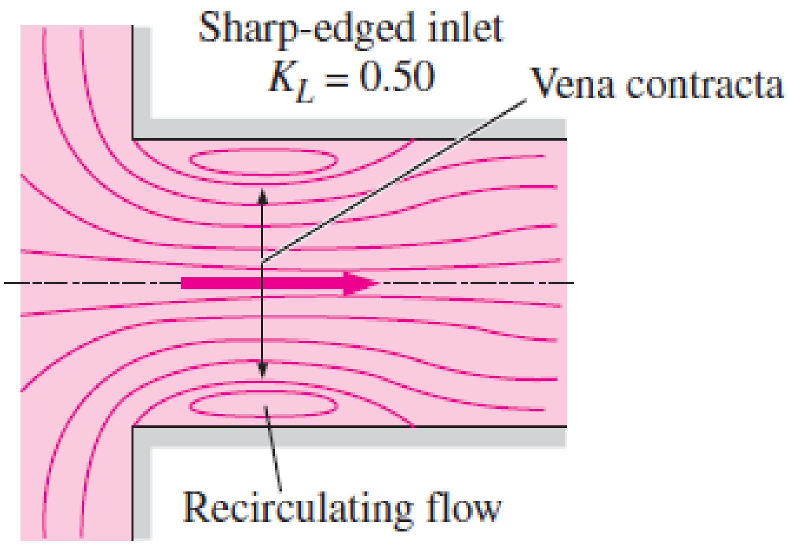
*Total head loss (general)*

$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}}$$

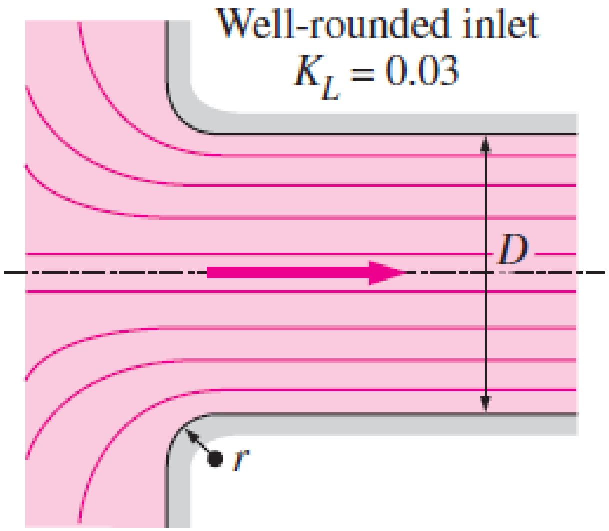
$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

*Total head loss (D = constant)*

$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$




The head loss at the inlet of a pipe is almost negligible for well-rounded inlets ($K_L = 0.03$ for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets.

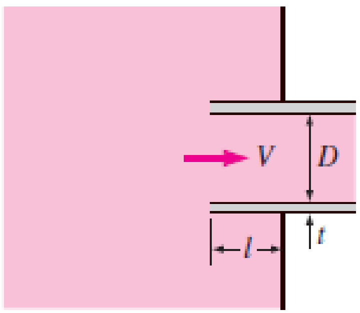


## TABLE 8–4

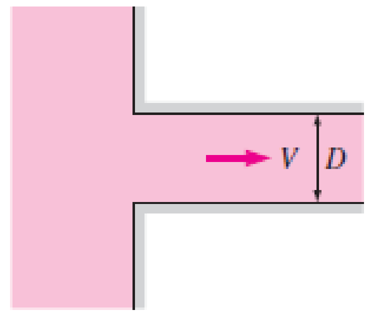
Loss coefficients $K_L$ of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where $V$ is the average velocity in the pipe that contains the component)*

*Pipe Inlet*

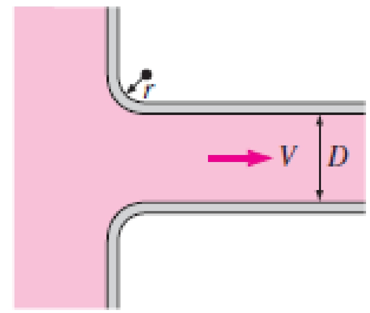
*Reentrant:* $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)




*Sharp-edged:* $K_L = 0.50$

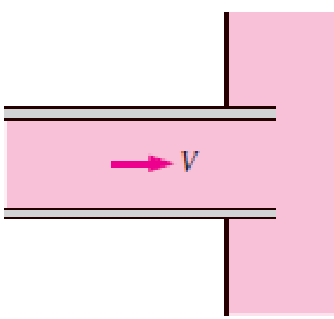



*Well-rounded* ($r/D > 0.2$): $K_L = 0.03$
*Slightly rounded* ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)

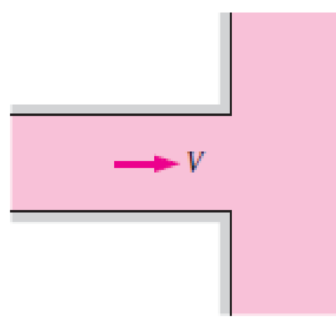



*Pipe Exit*

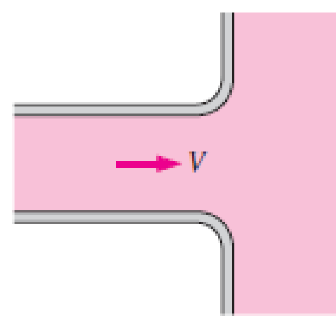
*Reentrant:* $K_L = \alpha$




*Sharp-edged:* $K_L = \alpha$



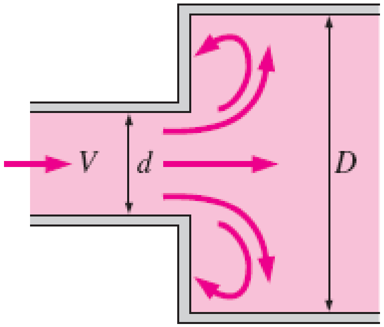

*Rounded:* $K_L = \alpha$



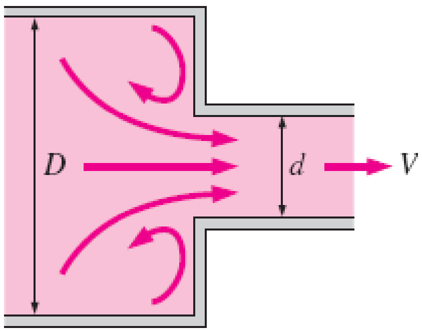

Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

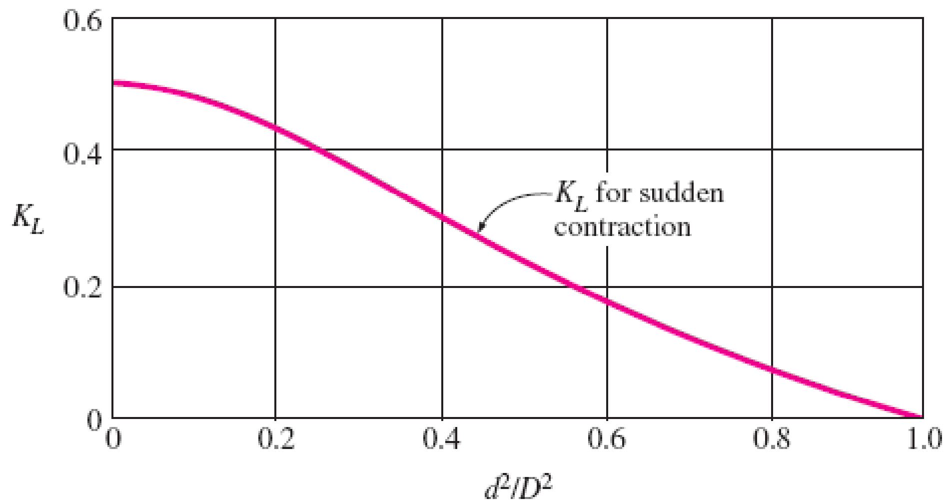
## Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

*Sudden expansion*: $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$




*Sudden contraction*: See chart.

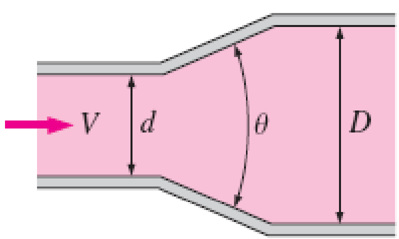



## Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

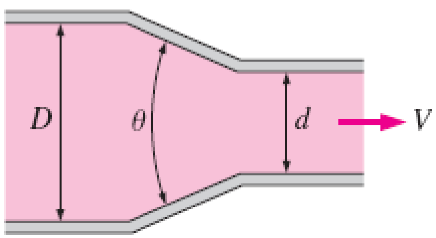
*Expansion* (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$



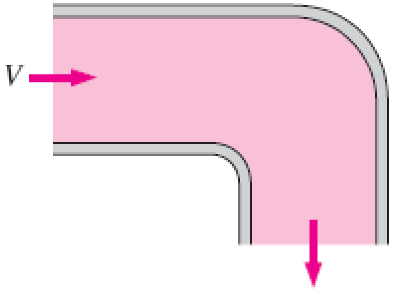

*Contraction*:

$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

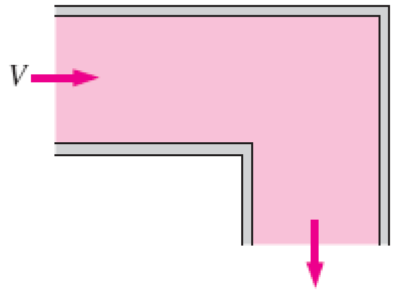


## Bends and Branches

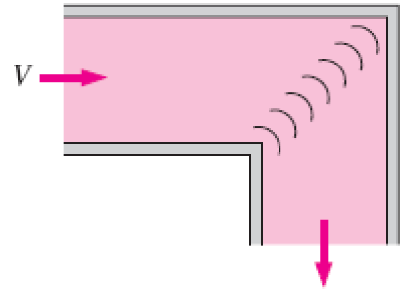
*90° smooth bend*:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



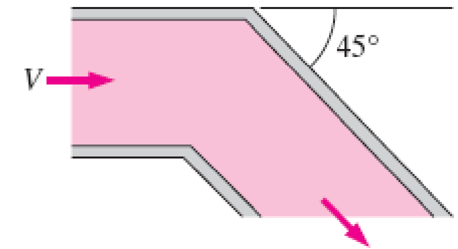

*90° miter bend*
(without vanes): $K_L = 1.1$



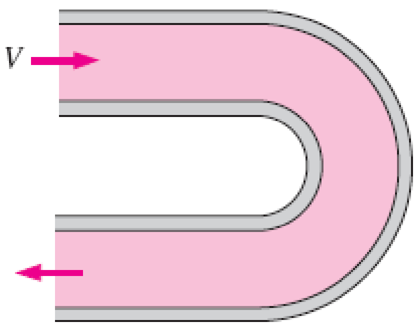

*90° miter bend*
(with vanes): $K_L = 0.2$



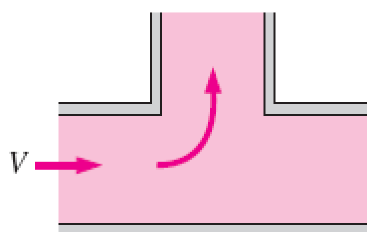

*45° threaded elbow*:
$K_L = 0.4$



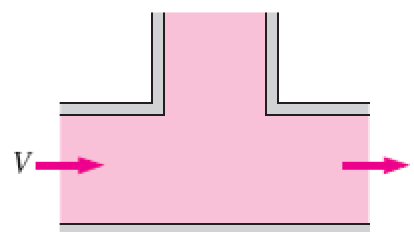

*180° return bend*:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



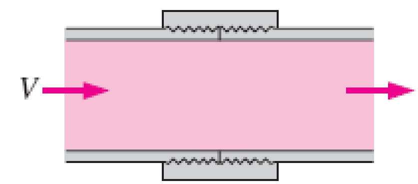

*Tee* (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$




*Tee* (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$




*Threaded union*:
$K_L = 0.08$

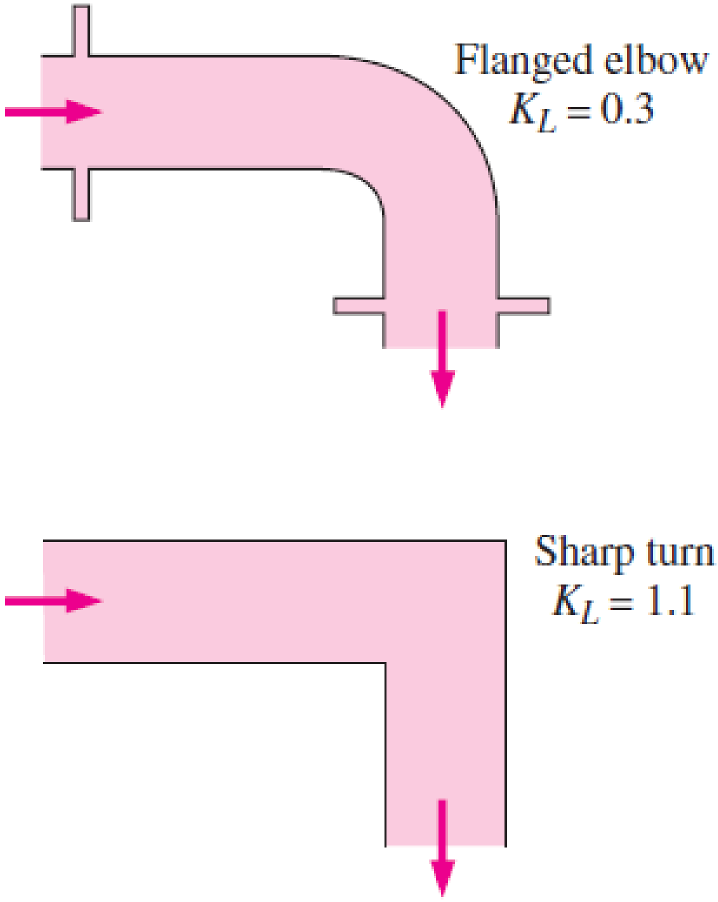



## Valves

*Globe valve*, fully open: $K_L = 10$
*Angle valve*, fully open: $K_L = 5$
*Ball valve*, fully open: $K_L = 0.05$
*Swing check valve*: $K_L = 2$

*Gate valve*, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$

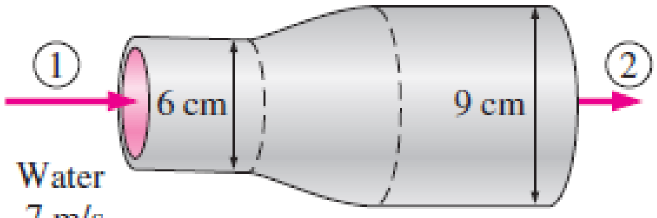
* These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from th
given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.




The losses during changes of direction can be minimized by making the turn “easy” on the fluid by using circular arcs instead of sharp turns.

**EXAMPLE 8–6 Head Loss and Pressure Rise during Gradual Expansion**

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig. 8–40). The walls of the expansion section are angled 10° from the axis. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.




**SOLUTION** A horizontal water pipe expands gradually into a larger-diameter pipe. The head loss and pressure after the expansion are to be determined.

***Assumptions*** **1** The flow is steady and incompressible. **2** The flow at sections 1 and 2 is fully developed and turbulent with $\alpha_1 = \alpha_2 \cong 1.06$.

***Properties*** We take the density of water to be $\rho = 1000\ \text{kg/m}^3$. The loss coefficient for a gradual expansion of total included angle $\theta = 20°$ and diameter ratio $d/D = 6/9$ is $K_L = 0.133$ (by interpolation using Table 8–4).

***Analysis*** Noting that the density of water remains constant, the downstream velocity of water is determined from conservation of mass to be

$$\dot m_1 = \dot m_2 \quad \rightarrow \quad \rho V_1 A_1 = \rho V_2 A_2 \quad \rightarrow \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1$$

$$V_2 = \frac{(0.06\ \text{m})^2}{(0.09\ \text{m})^2}(7\ \text{m/s}) = 3.11\ \text{m/s}$$

Then the irreversible head loss in the expansion section becomes

$$h_L = K_L \frac{V_1^2}{2g} = (0.133)\frac{(7\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{0.333\ m}$$

Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section is expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \cancel{z_1} + h_{\text{pump}, u} \overset{0}{\nearrow} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \cancel{z_2} + h_{\text{turbine}, e} \overset{0}{\nearrow} + h_L$$

or

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Solving for $P_2$ and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - g h_L\right\} = (150\ \text{kPa}) + (1000\ \text{kg/m}^3)$$

$$\times \left\{\frac{1.06(7\ \text{m/s})^2 - 1.06(3.11\ \text{m/s})^2}{2} - (9.81\ \text{m/s}^2)(0.333\ \text{m})\right\}$$

$$\times \left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$

$$= \mathbf{168\ kPa}$$

Therefore, despite the head (and pressure) loss, the pressure *increases* from 150 to 168 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the average flow velocity is decreased in the larger pipe.

***Discussion*** It is common knowledge that higher pressure upstream is necessary to cause flow, and it may come as a surprise to you that the downstream pressure has *increased* after the expansion, despite the loss. This is because the flow is driven by the sum of the three heads that comprise the total head (namely, the pressure head, velocity head, and elevation head). During flow expansion, the higher velocity head upstream is converted to pressure head downstream, and this increase outweighs the nonrecoverable head loss. Also, you may be tempted to solve this problem using the Bernoulli equation. Such a solution would ignore the head (and the associated pressure) loss and result in an incorrect higher pressure for the fluid downstream.

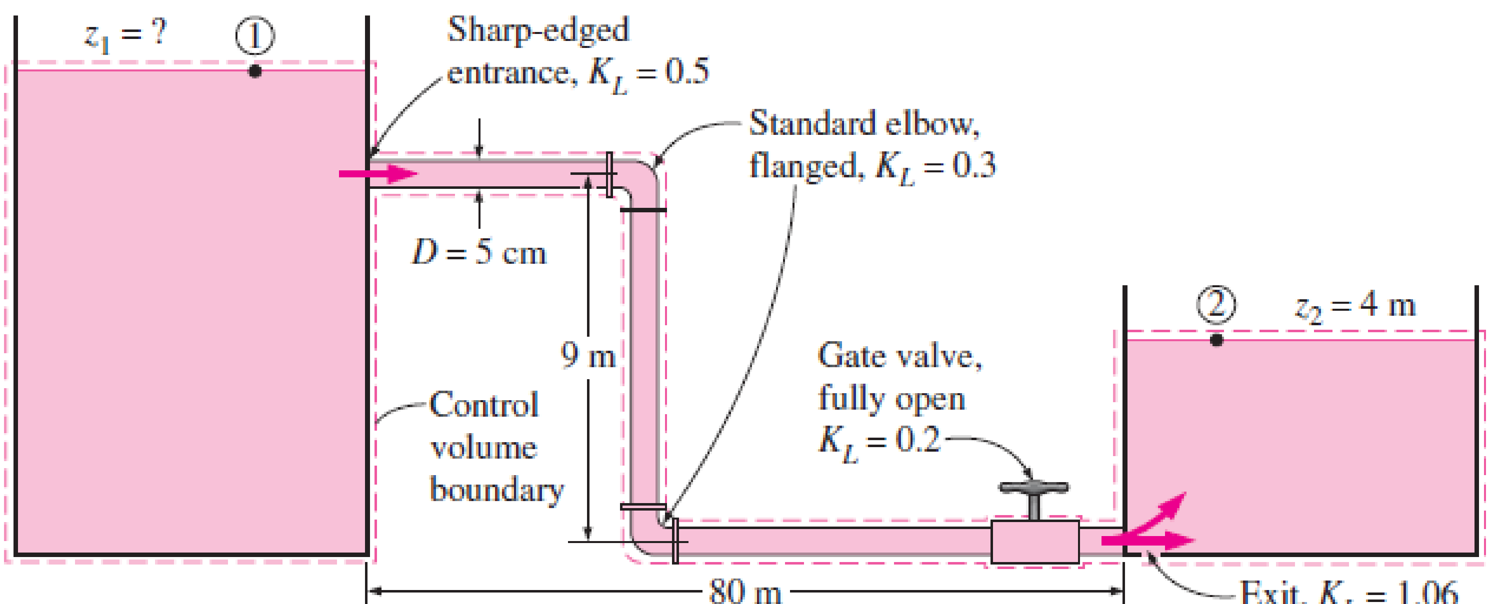
**EXAMPLE 8–8** **Gravity-Driven Water Flow in a Pipe**

Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in Fig. 8–48. Determine the elevation $z_1$ for a flow rate of 6 L/s.

**SOLUTION** The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

***Assumptions*** **1** The flow is steady and incompressible. **2** The elevations of the reservoirs remain constant. **3** There are no pumps or turbines in the line.

***Properties*** The density and dynamic viscosity of water at 10°C are $\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307 \times 10^{-3}\ \text{kg/m} \cdot \text{s}$. The roughness of cast iron pipe is $\varepsilon = 0.00026$ m (Table 8–2).



***Analysis*** The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are nearly zero ($V_1 \approx V_2 \approx 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g}^{\nearrow 0} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006 \text{ m}^3/\text{s}}{\pi(0.05 \text{ m})^2/4} = 3.06 \text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7 \text{ kg/m}^3)(3.06 \text{ m/s})(0.05 \text{ m})}{1.307 \times 10^{-3} \text{ kg/m} \cdot \text{s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor is determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\,\text{entrance}} + 2K_{L,\,\text{elbow}} + K_{L,\,\text{valve}} + K_{L,\,\text{exit}}$$
$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\,\frac{89\text{ m}}{0.05\text{ m}} + 2.36\right)\frac{(3.06\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 27.9\text{ m}$$

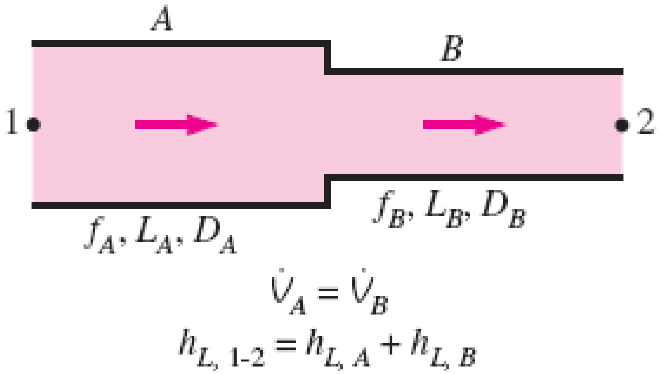
$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\text{ m}}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

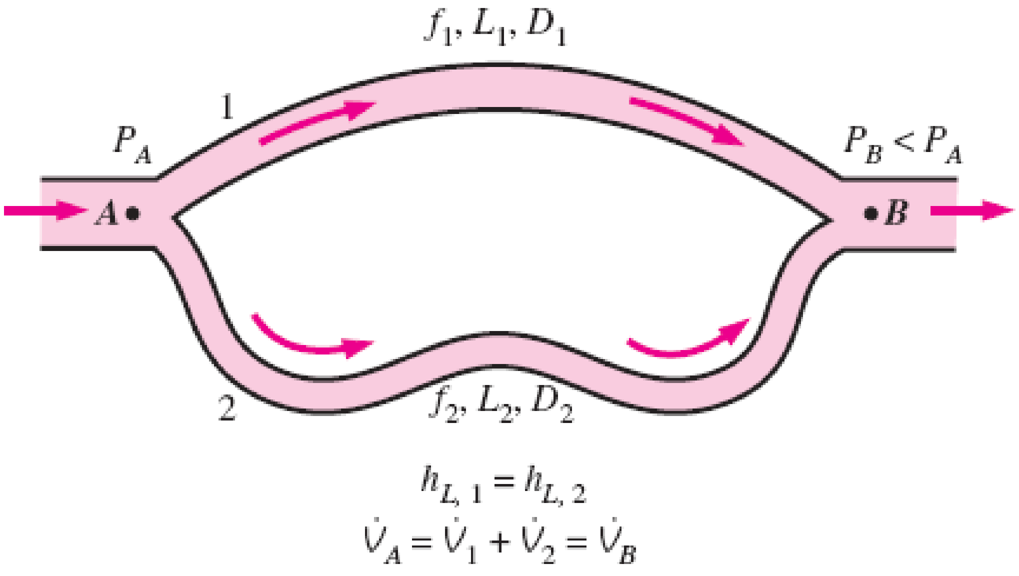
***Discussion*** Note that $fL/D = 56.1$ in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that at the same flow rate, the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes by smooth pipes such as those made of plastic.

# 14–7 ■ PIPING NETWORKS AND PUMP SELECTION

A piping network in an industrial facility.




For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.




For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same.

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

**The analysis of piping networks is based on two simple principles:**

1. ***Conservation of mass throughout the system must be satisfied*.** This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.
2. ***Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions*.** This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

# Summary

- Introduction
- Laminar and Turbulent Flows
  - ✓ Reynolds Number
- The Entrance Region
  - ✓ Entry Lengths
- Laminar Flow in Pipes
  - ✓ Pressure Drop and Head Loss
  - ✓ Effect of Gravity on Velocity and Flow Rate in Laminar Flow
  - ✓ Laminar Flow in Noncircular Pipes
- Turbulent Flow in Pipes
  - ✓ Turbulent Shear Stress
  - ✓ Turbulent Velocity Profile
  - ✓ The Moody Chart and the Colebrook Equation
  - ✓ Types of Fluid Flow Problems

- Minor Losses
- Piping Networks and Pump Selection
  - ✓ Serial and Parallel Pipes
  - ✓ Piping Systems with Pumps and Turbines

# Thank You…☺