MOTION OF PARTICLES TROUGH FLUIDS

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Mechanics of Particle Motion

- For a rigid particle moving through a fluid, there are 3 forces acting on the body

  - - The external force (gravitational or centrifugal force)
  - - The buoyant force (opposite but parallel direction to external force)
  - - The drag force (opposite direction to the particle motion)

Equation for One-dimensional Motion of Particle through Fluid

- Consider a particle of mass *m* moving through a fluid under the action of an external force $\boldsymbol{F_e}$. Let the velocity of the particle relative to the fluid be $\boldsymbol{u}$, let the buoyant force on the particle be $\boldsymbol{F_b}$ and let the drag be $\boldsymbol{F_D}$, then

$$m\frac{du}{dt} = F_e - F_b - F_D \qquad (1)$$

- **The external force** $\boldsymbol{(F_e)}$ **-** Expressed as a product of the **mass (m)** and the acceleration ($\boldsymbol{a_e}$) of the particle from this force

$$F_e = ma_e \qquad (2)$$

The buoyant force (F_b) – Based on Archimedes' law, the product of the mass of the fluid displaced by theparticle and the acceleration from the external force.

- The volume of the particle is $Vp = \frac{m}{\rho_p}$
- The mass of fluid displaced is $m = \frac{m}{\rho_p}\rho$

where ρ is the density of the fluid. The buoyant force is given by

$$F_b = \frac{m\rho a_e}{\rho_p} \quad (3)$$

The drag force (F_D)

$$F_D = \frac{C_D u^2 \rho A_p}{2} \quad (4)$$

where C_D is the drag coefficient, A_p is the projected area of the particle in the plane perpendicular to the flow direction.

- By substituting all the forces in the Eq. (1)

$$\frac{du}{dt} = a_e - \frac{\rho a_e}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = a_e \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \quad (5)$$

- Case 1 : Motion from gravitational force

$$a_e = g$$

$$\frac{du}{dt} = g \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \quad (6)$$

- Case 2 : Motion in a centrifugal field

$$a_e = r\omega^2$$

$$\frac{du}{dt} = r\omega^2 \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} \qquad (7)$$

- r = radius of path of particles
- ω = angular velocity, rad/s

- In this equation, *u* is the velocity of the particle relative to the fluid and is directed outwardly along a radius.

Terminal Velocity

- **In gravitational settling**, *g* is constant (9.81m/s2)
- The drag (C_D) always increases with velocity (u).
- The acceleration (a) decreases with time and approaches zero.
- The particle quickly reaches a constant velocity which is the maximum attainable under the circumstances.
- **This maximum settling velocity is called terminal velocity.**

$$\frac{du}{dt} = g\frac{\rho_{p-}\rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = 0 \qquad (8)$$

$$u_t = \sqrt{\frac{2g(\rho_{p-}\rho\ )m}{A_p \rho_p C_D \rho}} \qquad (9$$

- For **spherical particle** of diameter D_p moving through the fluid, the terminal velocity is given by

$$m = \frac{1}{6}\pi D_p^3 \rho_p \qquad A_p = \frac{1}{4}\pi D_p^2$$

- Substitution of $\boldsymbol{m}$ and $\boldsymbol{A_p}$ into the equation for $\boldsymbol{u_t}$ gives the equation for **gravity settling** of spheres

$$u_t = \sqrt{\frac{4 g D_p (\rho_p - \rho)}{3 C_D \rho}} \qquad (12)$$

Frequently used

- **In motion from a centrifugal force**, the velocity depends on the radius
- The acceleration is not constant if the particle is in motion with respect to the fluid.
- In many practical use of centrifugal force, is small ($\frac{du}{dt} = \sim 0$) thus, it can be neglected to give

$$\frac{du}{dt} = r\omega^2 \frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m} = 0 \quad (10)$$

$$u_t = \omega \sqrt{\frac{2r(\rho_p - \rho)m}{A_p \rho_p C_D \rho}} \quad (11)$$

Drag Coefficient

- Drag coefficient is a function of Reynolds number (N_{RE}). The drag curve applies only under restricted conditions:

  i). The particle must be a solid sphere;

  ii). The particle must be far from other particles and the vessel wall so that the flow pattern around the particle is not distorted;

  iii). It must be moving at its terminal velocity with respect to the fluid.

Reynolds Number

- Particle Reynolds Number

$$Re = \frac{u D_p \rho}{\mu}$$

u : velocity of fluid stream
D_p : diameter of the particle
ρ : density of fluid
μ : viscosity of fluid

- For Re < 1 (Stokes Law applied- laminar flow)

$$C_D = \frac{24}{Re_p}$$

Thus,

$$F_D = 3\pi\mu u_t D_p \qquad u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\mu}$$

- For 1000 < Re <200 000 (Newton's Law applied – turbulent flow)

$$C_D = 0.44$$

$$F_D = 0.055\pi D_p^2 u_t^2 \rho \qquad u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$$

- Newton's law applies to fairly large particles falling in gases or low viscosity fluids.

Criterion for settling regime

- To identify the range in which the motion of the particle lies, the velocity term is eliminated from the Reynolds number (Re) by substituting u_t from Stokes' law and Newton's law.
- Using Stoke's Law; $u_t = \dfrac{g D_p^2 (\rho_p - \rho)}{18\mu}$

$$Re = \frac{D_p u_t \rho}{\mu} = \frac{g \rho D_p^3 (\rho_p - \rho)}{18\mu^2}$$

- To determine the settling regime, a convenient criterion K is introduced.

$$K = D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

- Thus $Re = K^3/18$.
- Set $Re = 1$ and solving for K gives $K=2.6$.
- If $K < 2.6$ then Stokes' law applies.

- Using Newton's Law; $u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$
- Substitution by criterion K,

$$K = D_p\left[\frac{g\rho(\rho_p - \rho)}{\mu^2}\right]^{1/3}$$

- Thus, $Re = 1.75K^{1.5}$
- Set $Re = 1000$ and solving for K gives $K = 68.9$.
- Set $Re = 200{,}000$ and solving for K gives $K =$ 2,360.

THUS;

- Stokes' law range: $K < 2.6$

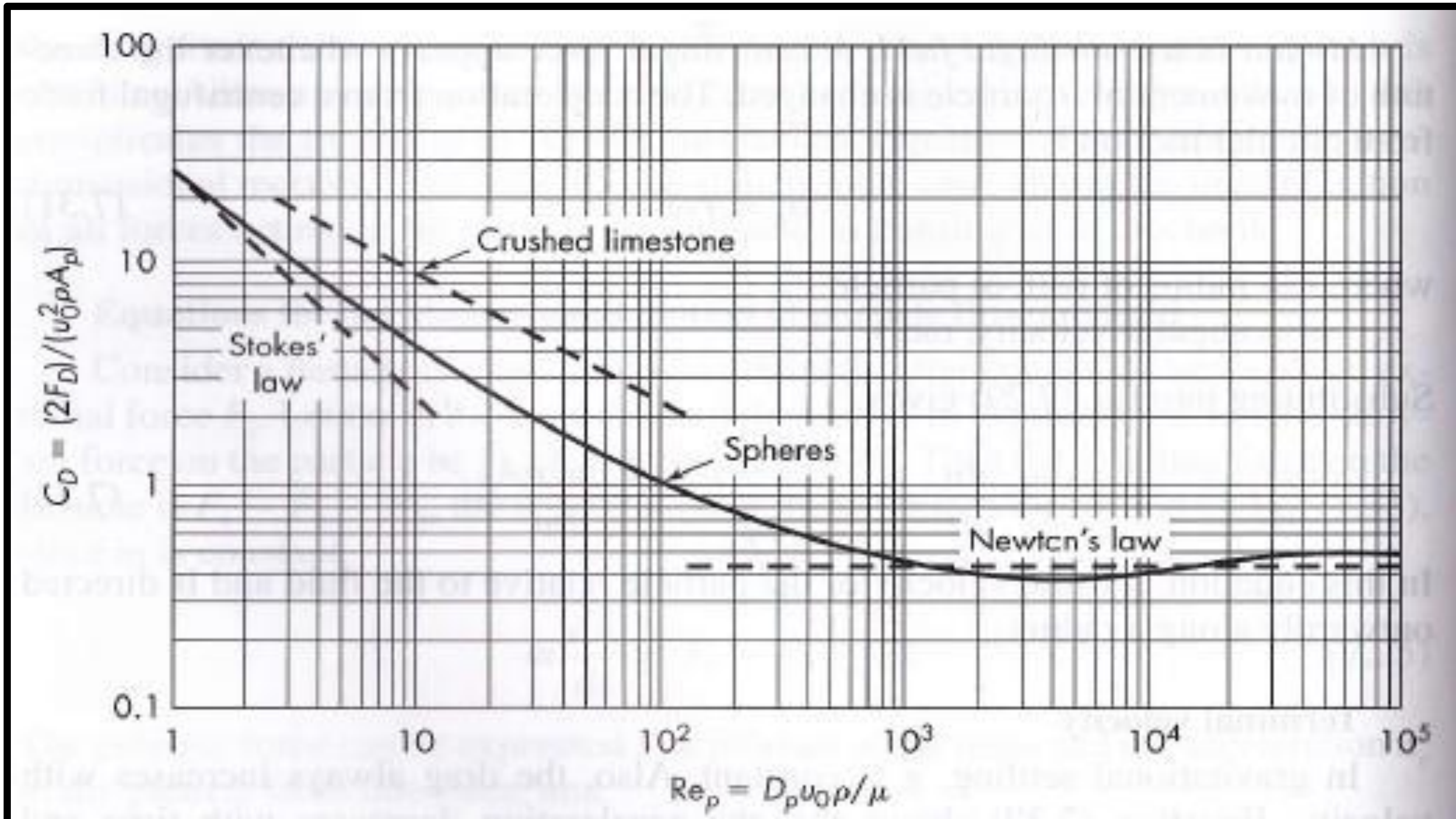
- Newton's law range: $68.9 < K < 2,360$

- Intermediate range : when $K > 2,360$ or $2.6 < K < 68.9$, u_t is found from;

$$u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$$

- using a value of C_D found by trial from the curve.

- **In general case,** the terminal velocity, u_t can be found by try and error after guessing *Re* to get an initial estimate of drag coefficient C_D .
- Normally for this case the particle diameter *Dp* is known



Drag coefficients (C_D) for spheres and irregular particles

Example 1

a. Estimate the terminal velocity for 80- to 100-mesh particles of limestone (ρ_p = 2800 kg/m^3) falling in water at 30° C.

b. How much bigger would be the velocity in centrifugal separator when the acceleration is 50g?

Solution 1

(a) From Appendix:

D_p for 100-mesh = 0.147 mm

D_p for 80-mesh = 0.175 mm

Average diameter $\bar{D}_p$ = 0.161 mm

$\mu = 0.801$ cP and $\rho = 62.16$ lb/ft^3 or 995.7 kg/m^3.

To find which settling law applies, calculate criterion *K*:

$$K = D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

$$K = 0.161 \times 10^{-3} \left[\frac{9.80665 \times 995.7(2{,}800 - 995.7)}{(0.801 \times 10^{-3})^2} \right]^{1/3}$$

$$= 4.86$$

This is slightly above Stoke's law range.

Assume $Re_p = 5;$ $then\ from\ the\ Fig.,C_D$ $\cong$14, and,

$$u_t = \sqrt{\frac{4gD_p(\rho_p - \rho)}{3C_D\rho}} \qquad (12)$$

$$u_t = \left[\frac{4 \times 9.80665(2{,}800 - 995.7)(0.161 \times 10^{-3})}{3 \times 14 \times 995.7}\right]^{1/2}$$
$$= 0.0165 \text{ m/s}$$

Check:

$$Re = \frac{uD_p\rho}{\mu}$$

$$\text{Re}_p = \frac{0.161 \times 10^{-3} \times 0.0165 \times 995.7}{0.801 \times 10^{-3}} = 3.30$$

Since C_D at Re_p = 3.30 is greater than 14, the revised u_t and Re_p will be less than the above values, so guess a lower value of Re_p .

Guess

$$Re_p = 2.5 \qquad C_D \cong 20$$

$$u_t = 0.0165\left(\tfrac{14}{20}\right)^{0.5} = 0.0138\ \text{m/s}$$

$$Re_p = 3.30\left(\frac{0.0138}{0.0165}\right) = 2.76$$

This is close enough to the value of 2.5, and

$$u_t \cong 0.014\ \text{m/s}$$

b. Using $a_e = 50g$ in place of g, since only the acceleration changes, $K = 4.86 \times 50^{1/3} = 17.90$. This is still in the intermediate settling range. Estimate $Re_p = 40$, from the Fig., $C_d = 4.1$ and

$$u_t = \left[\frac{4 \times 9.80665 \times 50(2,800 - 995.7)(0.161 \times 10^{-3})}{3 \times 4.1 \times 995.7}\right]^{1/2}$$

$$= 0.216 \text{ m/s}$$

Check:

$$\mathrm{Re}_p = \frac{0.161 \times 10^{-3} \times 0.216 \times 995.7}{0.801 \times 10^{-3}}$$

$$= 43, \text{ close to } 40$$

$$u_t \cong 0.22 \text{ m/s}$$

The calculated terminal velocities are about 30 percent less than for a sphere of the same size

Example 2

Oil droplets having diameter of 20 μm are to be settled from air with a density of 1.137kg/m^3 and viscosity of 1.9 x 10-5 Pa.s at. Meanwhile, the density of the oil is 900 kg/m^3. Calculate the terminal velocity of the droplets if the droplets is assumed to be a rigid sphere.

Solution: The solution is trial and error since the velocity is unknown. Hence, CD cannot be directly determined. The Reynolds number is as follows:

$$N_{Re} = \frac{D_p v_t \rho}{\mu} = \frac{(2.0 \times 10^{-5})(v_t)(1.137)}{1.90 \times 10^{-5}} = 1.197 v_t$$

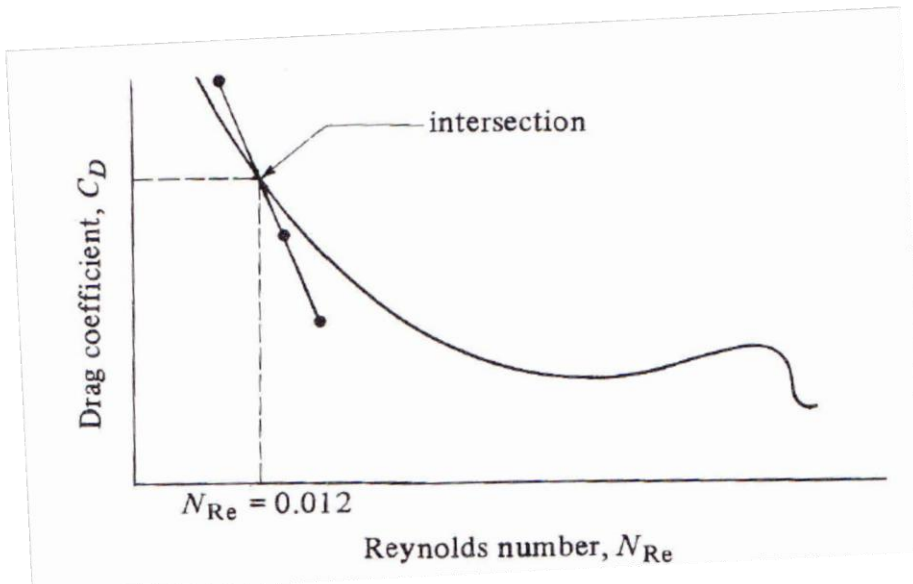
For the first trial, assume that $v_t = 0.305$ m/s. Then $N_{Re} = 1.197(0.305) = 0.365$. Solving for C_D:

$$v_t = \sqrt{\frac{4(\rho_p - \rho) g D_p}{3 C_D \rho}} = \sqrt{\frac{4(900 - 1.137)(9.8066)(2.0 \times 10^{-5})}{(3) C_D (1.137)}}$$

$$C_D = \frac{0.2067}{v_t^2}$$

Using $v_t = 0.305$ m/s, $C_D = 0.2067/(0.305)^2 = 2.22$.

Assuming that $v_t = 0.0305$ m/s, $N_{Re} = 0.0365$ and $C_D = 222$. For the third trial, assuming that $v_t = 0.00305$ m/s, $N_{Re} = 0.00365$ and $C_D = 22200$. These three values calculated for N_{Re} and C_D are plotted on a graph.



It can be shown that the line through these points is a straight line. The intersection of this line and the drag-coefficient correlation line is the solution to the problem at $N_{Re} = 0.012$. The velocity can be calculated from the Reynolds number:

$$N_{Re} = 0.012 = 1.197 v_t$$

$$v_t = 0.0100 \text{ m/s } (0.0328 \text{ ft/s})$$

The particle is in the Reynolds number range less than 1, which is the laminar Stokes'law region. Alternatively, the velocity can be calculated as follows:

$$v_t = \frac{9.8066(2.0 \times 10^{-5})^2(900 - 1.137)}{18(1.90 \times 10^{-5})} = 0.0103 \text{ m/s}$$