Pertemuan ke-5

- **Application of Bernoulli's Equation in**

  Venturi meter, orifice meter and pitot tube.

- **Introduce conservation of momentum principle**

  Momentum transfer: the concept of balance of forces.

Bernoulli's equation

As all of these elements of the equation have units of length, they are often referred to as the following:

$$\frac{P}{\rho g} + z + \frac{u^2}{2g} = H$$

Total head

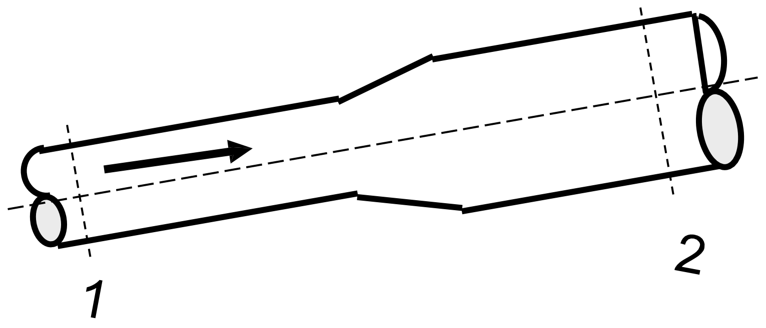
Velocity head

Pressure head

Elevation head / Gravity head

- In fluid dynamics, ***head*** is a concept that relates the energy in an incompressible fluid to the height of an equivalent static column of that fluid.
- Head is equal to the fluid's energy per unit weight.

We can apply it between two points, 1 and 2, on the streamline in the Fig.



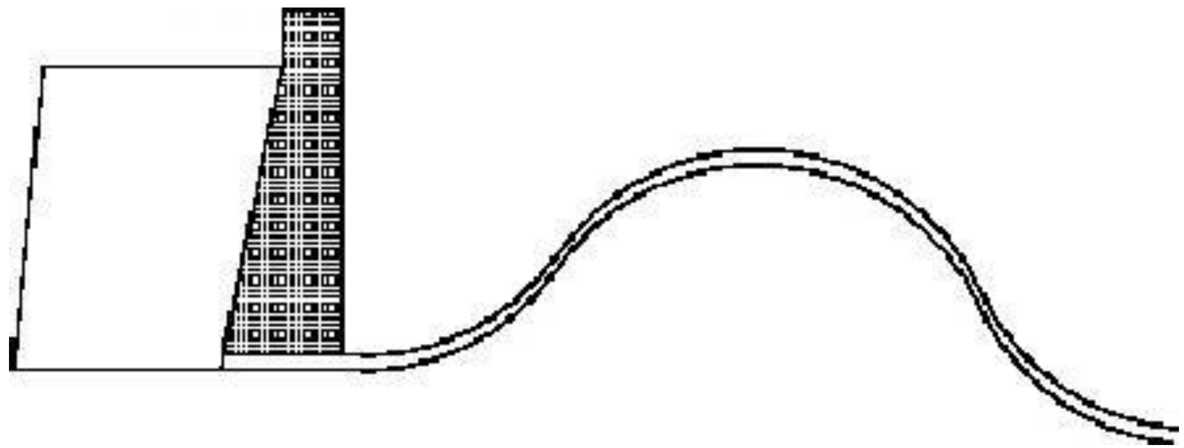
Total energy per unit weight at 1 = Total energy per unit weight at 2

or total head at 1 = total head at 2

or $$\frac{P_1}{\rho g} + z_1 + \frac{u_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{u_2^2}{2g} = H$$

Pressure head, velocity head, potential head and total head

A useful method of analyzing the flow is to show the pressures graphically



Consider the reservoir below feeding a pipe which changes diameter and rises (in reality it may have to pass over a hill) before falling to its final level

What will be the total head if pipe nozzle is closed

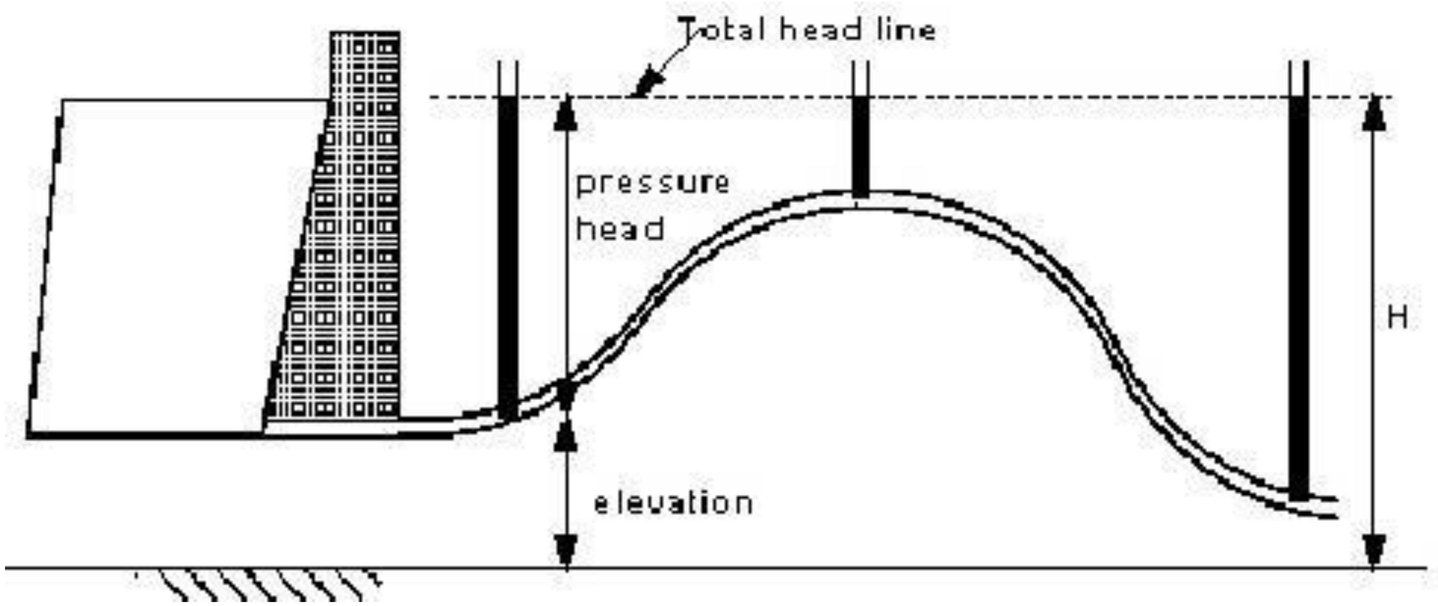


Fig. Piezometer levels with zero velocity

Since nozzle is closed the velocity of fluid u =0 in each point, so from Bernounill equation we can write,

$$\frac{P}{\rho g} + z = H$$

What would happen to the levels in the piezometers (pressure heads) if the water is flowing with velocity =u?

We know from Bernuonill principle that as velocity increases so pressure falls …

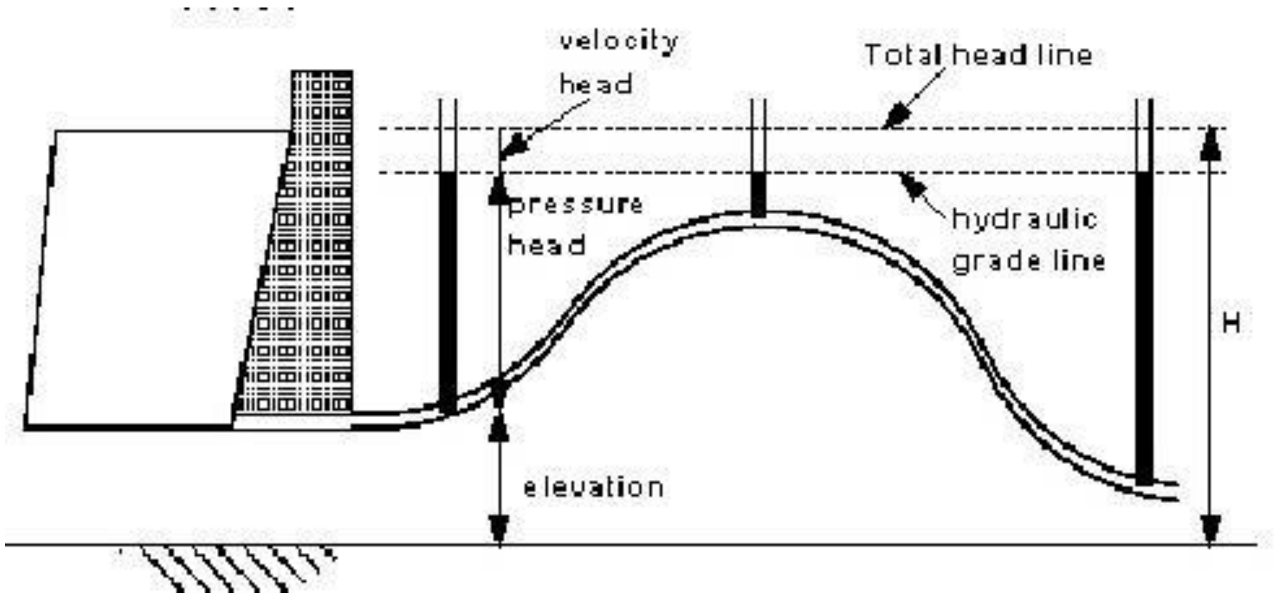


Fig. Piezometer levels and velocity heads with fluid flowing in constant diameter pipe

Since the diameter of the pipe constant, the velocity is equal in each point

We can Write,
$$\frac{P}{\rho g} + z + \frac{u^2}{2g} = H$$

What would happen if the pipe are not of constant diameter?

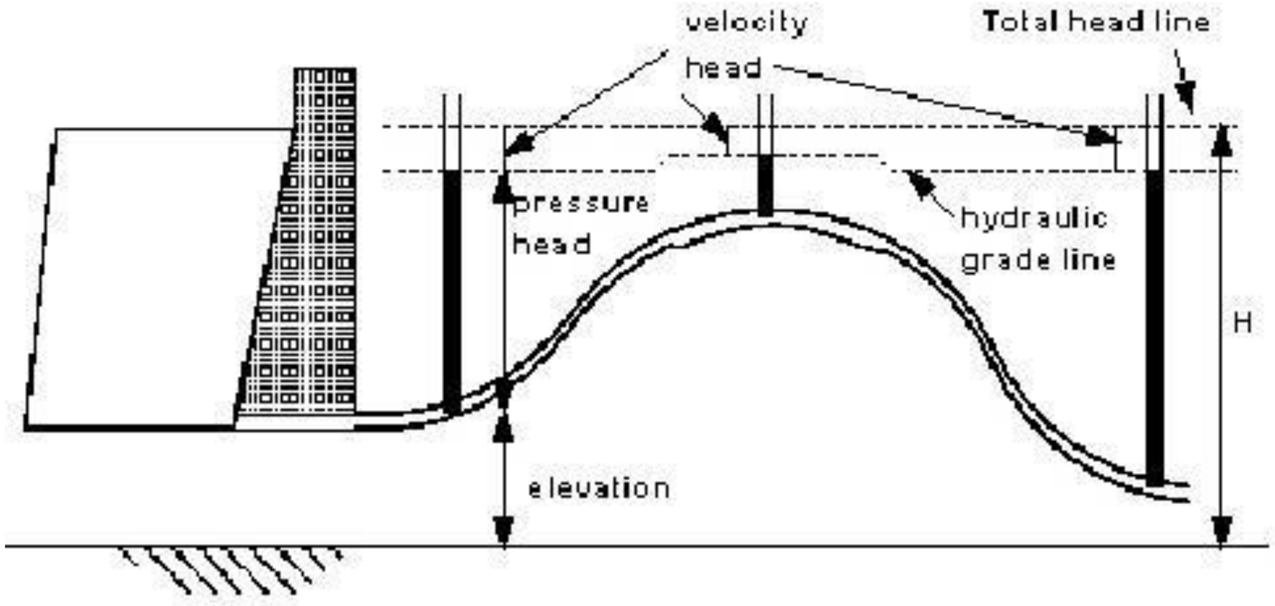
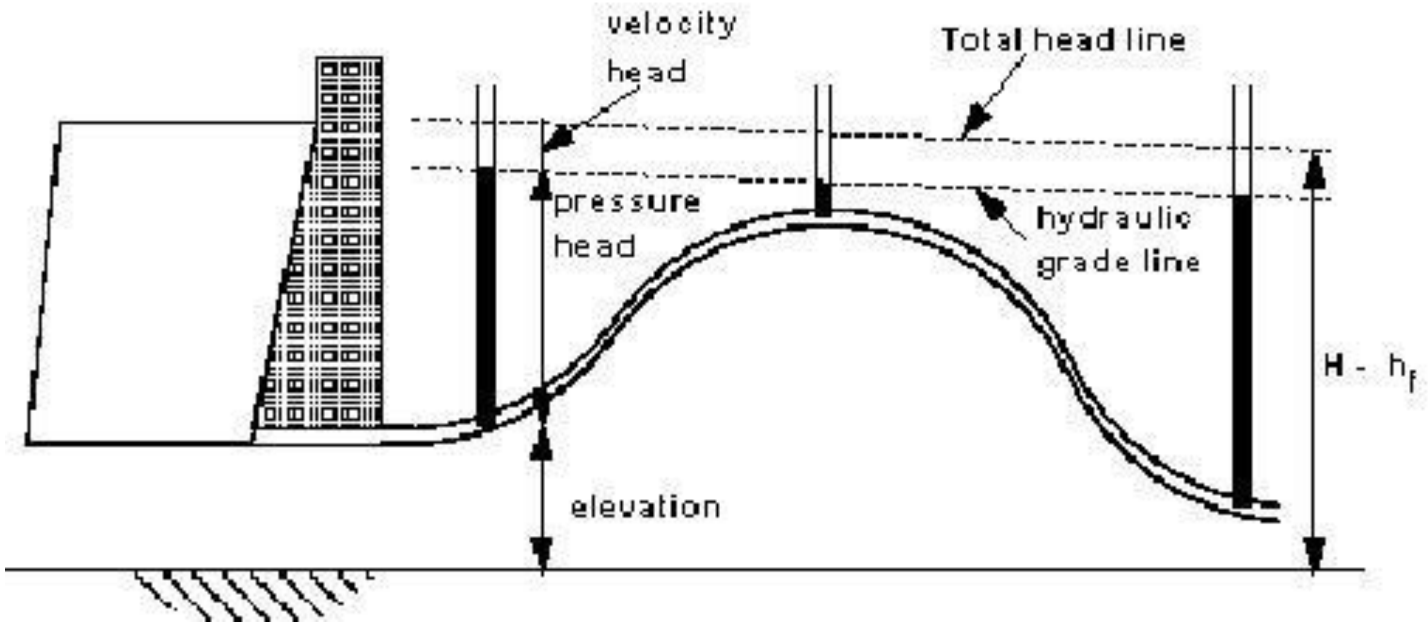


Fig. Piezometer levels and velocity heads with fluid flowing in varying diameter pipe

$$\frac{P}{\rho g} + z + \frac{u^2}{2g} = H$$

Energy losses due to friction

In a real pipe line there are energy losses due to friction - these must be taken into account as they can be very significant. How would the pressure and hydraulic grade lines change with friction?



$$\frac{P_1}{\rho g} + z_1 + \frac{u_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{u_2}{2g} + h_f$$

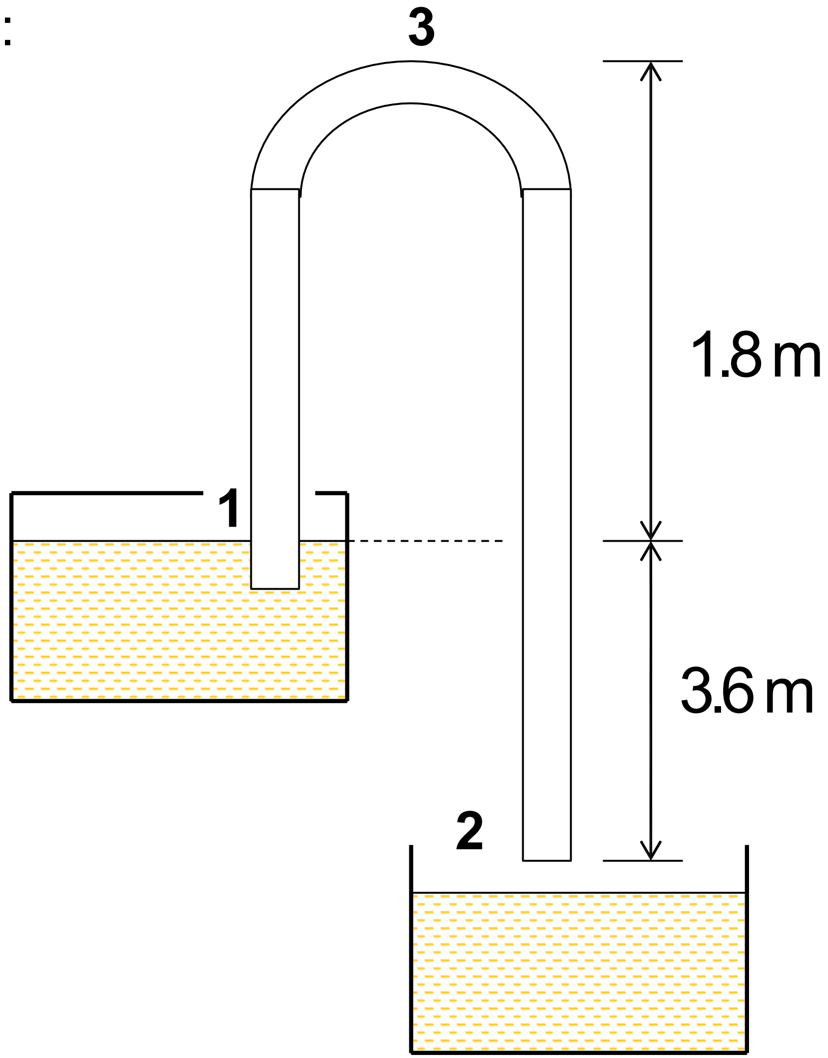
Here, h_f is head loss due to friction

Class example 1

A siphon has a uniform circular bore of 75 mm diameter and consist of a bent pipe with its crest 1.8 m above water level discharging into the atmosphere at a level 3.6 m below water level. If the atmospheric pressure is equivalent of 10 m of water, determine:

1. The outlet velocity
2. The flow rate out
3. The absolute pressure at crest level.

Neglect the losses due to friction.



- Assumption:
  - No work interaction.
  - The pipe is frictionless
  - Tank is very large that the velocity of gas at point 1 is relatively small, $V_1 \cong 0$
  - No velocity change at point 1 and 3 → $V_3 \cong 0$
  - Pressure at point 1 and 2 = P_{atm} = 0 kPag
  - Steady flow

To be solved in class

Applications of the Bernoulli Equation

The Bernoulli equation can be applied to a great many situations not just the pipe flow we have been considering up to now. it has application to flow measurement from tanks, within pipes as well as in open channels

- Orifice Plate
- Venturi Tube
- Pitot Tube
- Rota meter
- Target mater
- Nozzle
- Elbow Meter
- Bypass Meter

Learning Outcome

- At the end of this topic you shall be able to:
  - Derive relationship between pressure drop and velocity.
  - Determine flow rate for using devices such as venturi meter, orifice meter and pitot tube.

Pitot tube

- A device that can be used to measure flow rates of fluid.
- Based on the Bernoulli's principle, and using the concept of stagnation pressure.
- Stagnation pressure = static pressure + dynamic pressure

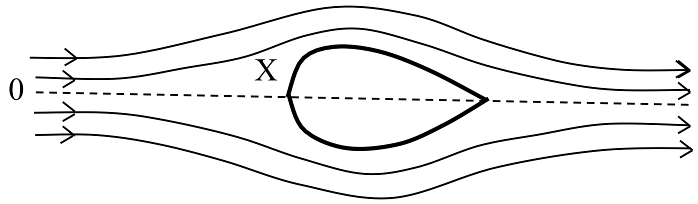
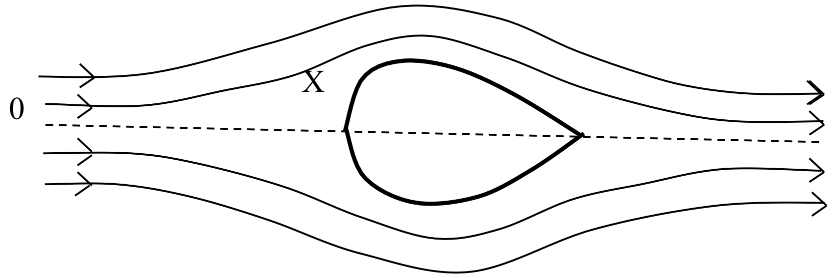


Figure :
Streamlines passing a non-rotating obstacle

- A point in a fluid stream where the velocity is reduced to zero is known as a *stagnation point.*
- Any non-rotating obstacle placed in the stream produces a stagnation point next to its upstream surface.
- The velocity at X is zero: X is a stagnation point.

- By Bernoulli's equation the quantity $p + \frac{1}{2}\rho V^2 + \rho g z$ is constant along a streamline for the steady frictionless flow of a fluid of constant density.
- If the velocity V at a particular point is brought to zero the pressure there is increased from p to $p + \frac{1}{2}\rho V^2$.
- For a constant-density fluid the quantity $p + \frac{1}{2}\rho V^2$ is therefore known as the *stagnation pressure* of that streamline while $\frac{1}{2}\rho V^2$ – that part of the stagnation pressure due to the motion – is termed the *dynamic pressure.*
- A manometer connected to the point X would record the stagnation pressure, and if the *static* pressure p were also known $\frac{1}{2}\rho V^2$ could be obtained by subtraction, and hence V calculated.



- Measurement of the static pressure may be made at the boundary of the flow, as illustrated in **(a)**, provided that the axis of the piezometer is perpendicular to the boundary and the connection is smooth and that the streamlines adjacent to it are not curved
- A tube projecting into the flow (Tube **c**) does not give a satisfactory reading because the fluid is accelerating round the end of the tube.

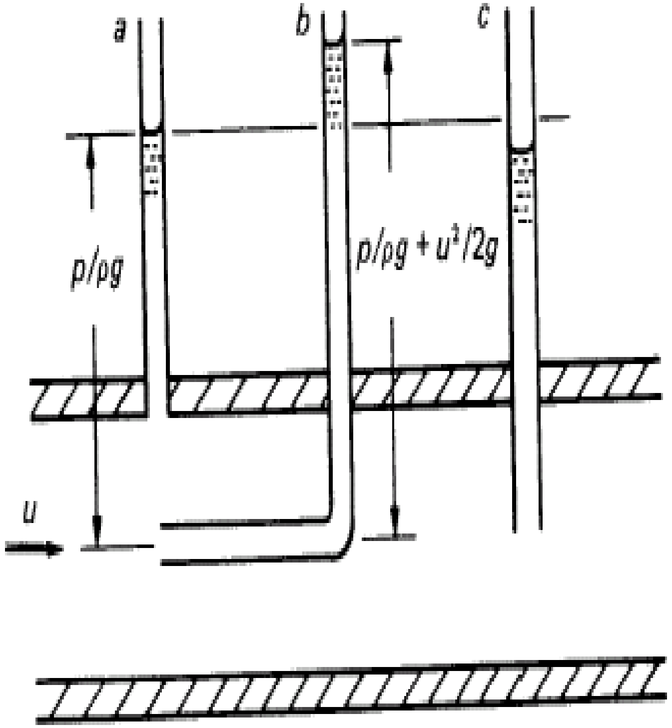


Fig. Piezometers connected to a pipe

- Two piezometers, one as normal and one as a Pitot tube within the pipe can be used in an arrangement shown below to measure velocity of flow.
- We can write,

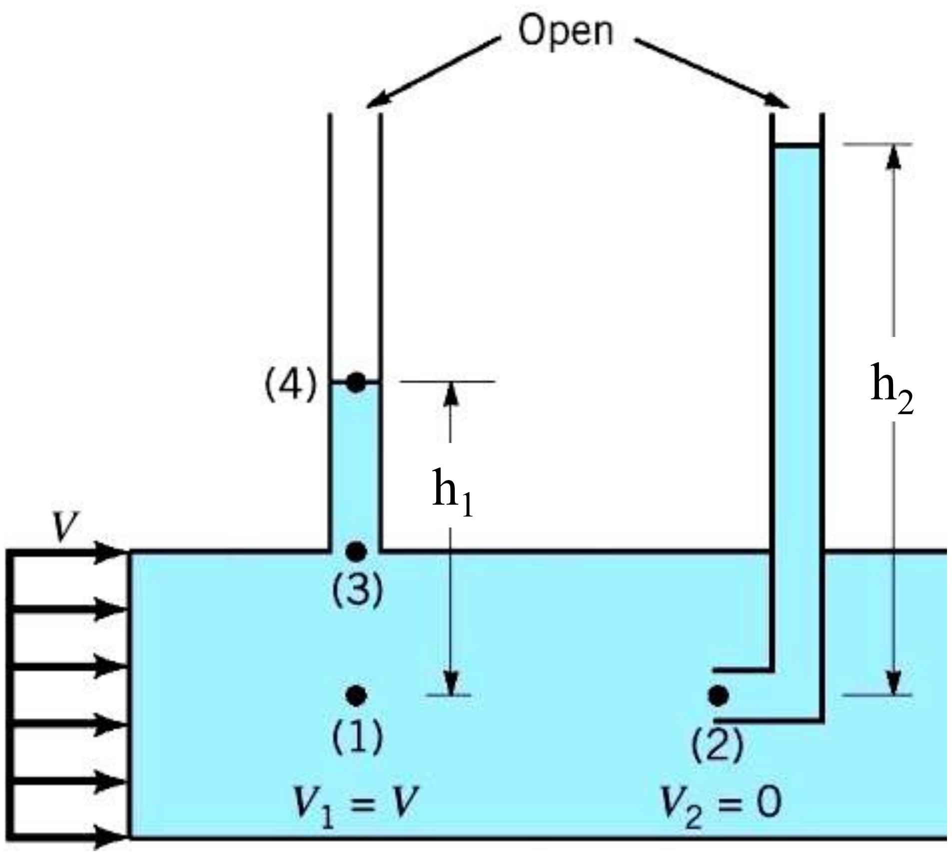
$$p_2 = p_1 + \frac{1}{2}\rho V_1^2$$

$$\boxed{V_1 = \sqrt{\frac{2\Delta P}{\rho}}}$$

Above equation can rite,

$$\rho g h_2 = \rho g h_1 + \frac{1}{2}\rho V_1^2$$

$$V1 = \sqrt{2g(h_2 - h_1)}$$



Class Example 2

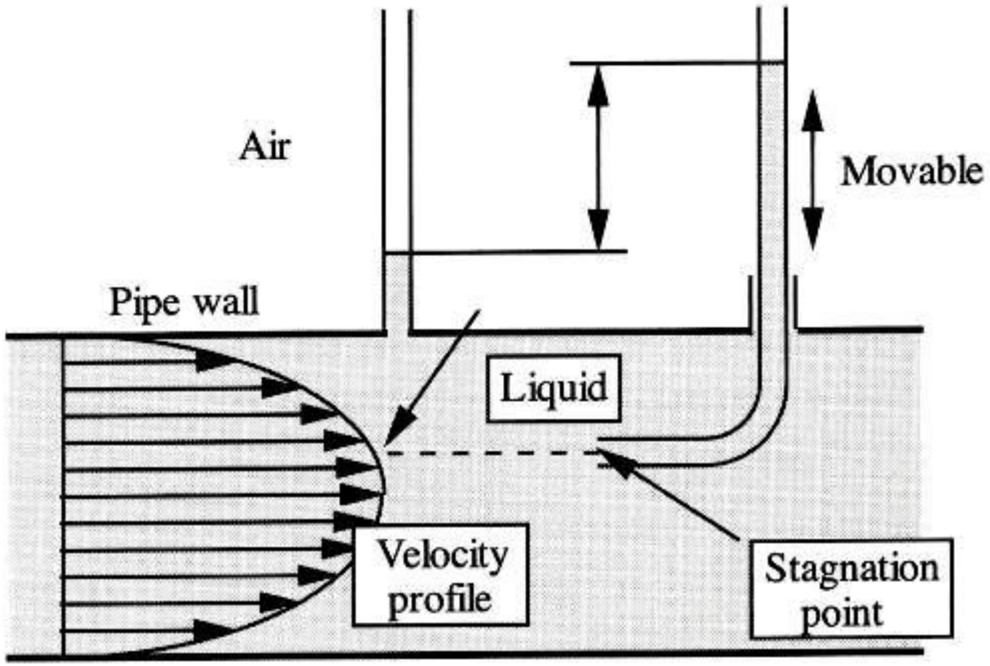
Air is flowing in a duct. The pressure-difference gage attached to the pitot-static tube indicates a difference of 200 Pa. what is the air velocity? (ρ_{air} = 1.20 kg/m^3)

- Solution:

$$V_1 = \sqrt{2\,\frac{\Delta P}{\rho}} = \sqrt{\frac{2 \times 200}{1.2}} = 18.28\,\frac{m}{s}$$

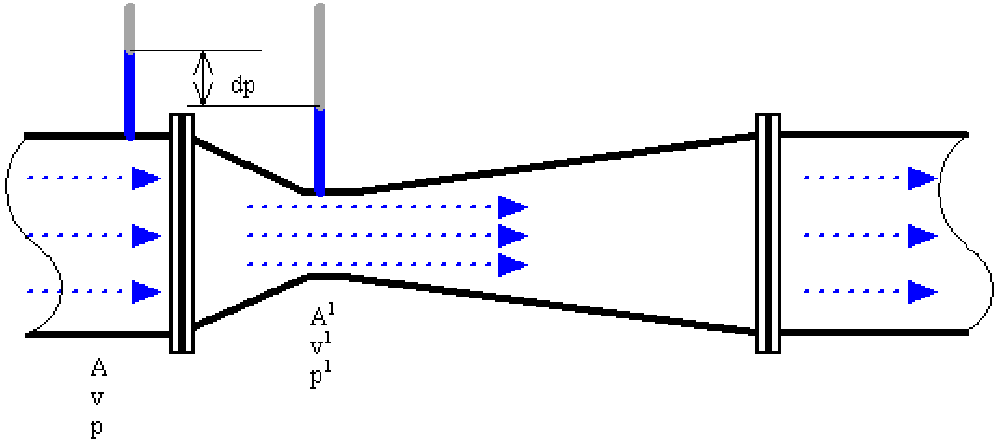
Limitation of Pitot Tube

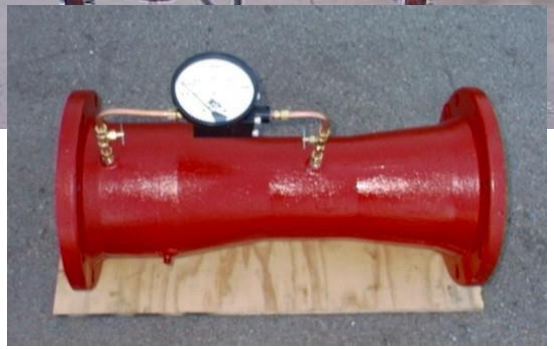
- The primary disadvantage of Pitot tube is that it must be aligned with the flow direction, which is unknown.



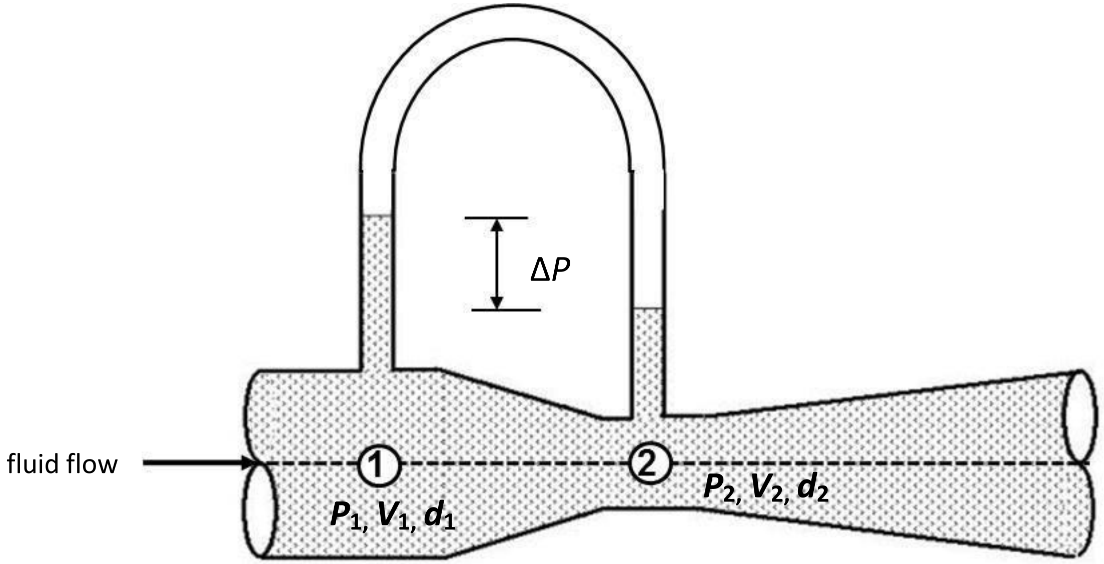
Venturi meter

- The Venturi meter is a device for measuring discharge in a pipe.
- It consists of a rapidly converging section, which increases the velocity of flow and hence reduces the pressure.
- It then returns to the original dimensions of the pipe by a gently diverging ‘diffuser’ section. By measuring the pressure differences the discharge can be calculated.
- This is a particularly accurate method of flow measurement as energy losses are very small.





Venturi meter



Applying Bernoulli Equation between (1) and (2), and using continuity equation will give :

$$Q_{ideal} = A_1 \sqrt{\frac{2\left(\frac{p_1 - p_2}{\rho}\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

To get the actual discharge, taking into consideration of losses due to friction, a coefficient of discharge, C_d, is introduced.

$$Q_{actual} = C_d \, Q_{ideal}$$

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2} = 0$$

$$\frac{P1-P2}{\rho} + g\,(Z1 - Z2) + \frac{U1^2-U2^2}{2} = 0$$

A1U1 = A2 U2

U2= U1 A1/A2

$$\frac{P1-P2}{\rho} + \frac{U1^2-U1^2A1^2/A2^2}{2} = 0$$

$$-\frac{U1^2-U1^2A1^2/A2^2}{2} = \frac{P1-P2}{\rho}$$

$$-\frac{U1^2 - U1^2 A1^2/A2^2}{2} = \frac{P1-P2}{\rho}$$

$$-\frac{U1^2\left(1-\frac{A1^2}{A2^2}\right)}{2} = \frac{P1-P2}{\rho}$$

$$-U1^2\left(1-\frac{A1^2}{A2^2}\right) = \frac{2(P1-P2)}{\rho}$$

$$\mathrm{U1} = \sqrt{\frac{2(P1-P2)}{\rho}} \times \frac{1}{\sqrt{\frac{A1^2}{A2^2}-1}} \quad \text{atau} \quad U2 = \sqrt{\frac{2(P1-P2)}{\rho}} \times \frac{1}{\sqrt{1-\left(\frac{A2^2}{A1^2}\right)}}$$

Class Example 3

The venturi meter has water flowing through it. The pressure difference $P_1 - P_2$ is 1 psi. The diameter at point 1 is 1 ft, and at point 2 is 0.5 ft. Determine the velocity through this meter.

- Assumption: Water → incompressible fluid, density constant (ρ = 62.4 lb_m/m^3)

$$V_2 = \sqrt{\frac{\frac{2(P_1 - P_2)}{\rho}}{\left(1 - \frac{A_2^{\,2}}{A_1^{\,2}}\right)}} = \sqrt{\frac{\frac{2\left(1\frac{lbf}{in^2} \times \frac{144\,in^2}{ft^2}\right)}{62.4\frac{lbm}{ft^3}} \times \frac{32.2\ lbm \cdot ft}{lbf \cdot s^2}}{\left(1 - \frac{0.5^2}{1^2}\right)}}$$

$$= 12.7\frac{ft}{s}$$

Class Example 4

A Venturi meter with an entrance diameter of 0.3 m and a throat diameter of 0.2 m is used to measure the volume of gas flowing through a pipe. The discharge coefficient of the meter is 0.96. Assuming the specific weight of the gas to be constant at 19.62 N/m^3, calculate the volume flowing when the pressure difference between the entrance and the throat is measured as 0.06 m on a water U-tube manometer.

What we know from the problem statement :

$\rho_g g = 19.62\ N/m^2$

$C_d = 0.96$

$d_1 = 0.3m$

$d_2 = 0.2m$

$V_1 = Q/0.0707$ $\qquad$ $V_2 = Q/0.0314$

Calculate Q:

For the manometer :

$$P_1 + \rho_g g z_1 = P_2 + \rho_g g(z_2 - R_P) + \rho_w g R_P$$

$$P_1 - P_2 = 19.62(z_2 - z_1) + 587.423 \qquad \text{--- (1)}$$

For the Venturi meter :

$$\frac{P_1}{\rho_g g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho_g g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = 19.62(z_2 - z_1) + 0.803 V_2^2 \qquad \text{--- (2)}$$

Combining (1) and (2) :

$$0.803 V_2^2 = 587.423$$

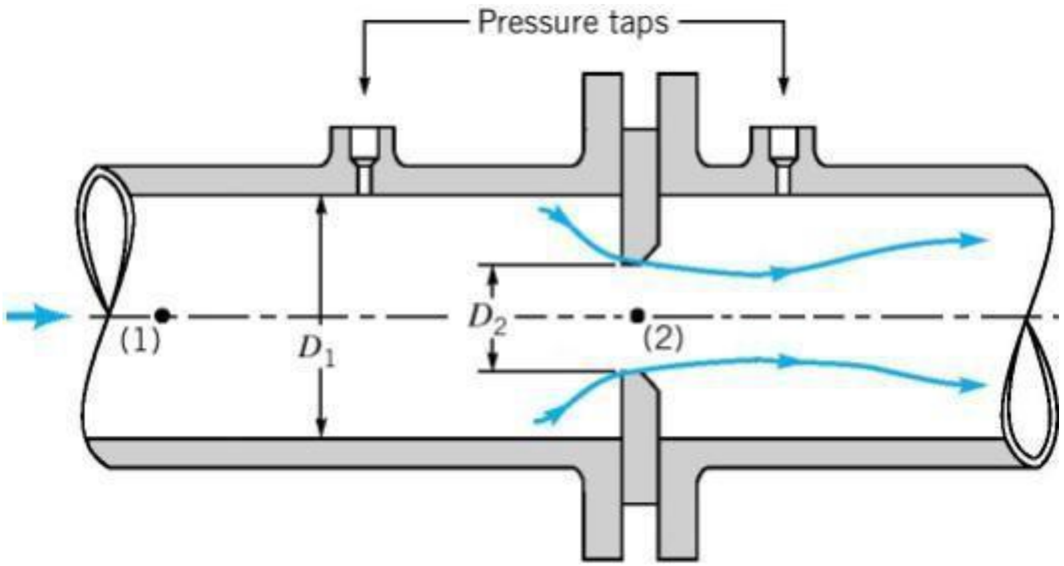
$$V_{2\,ideal} = 27.047\ m/s$$

$$Q_{ideal} = 27.047 \times \pi \left(\frac{0.2}{2}\right)^2 = 0.85\ m^3/s$$

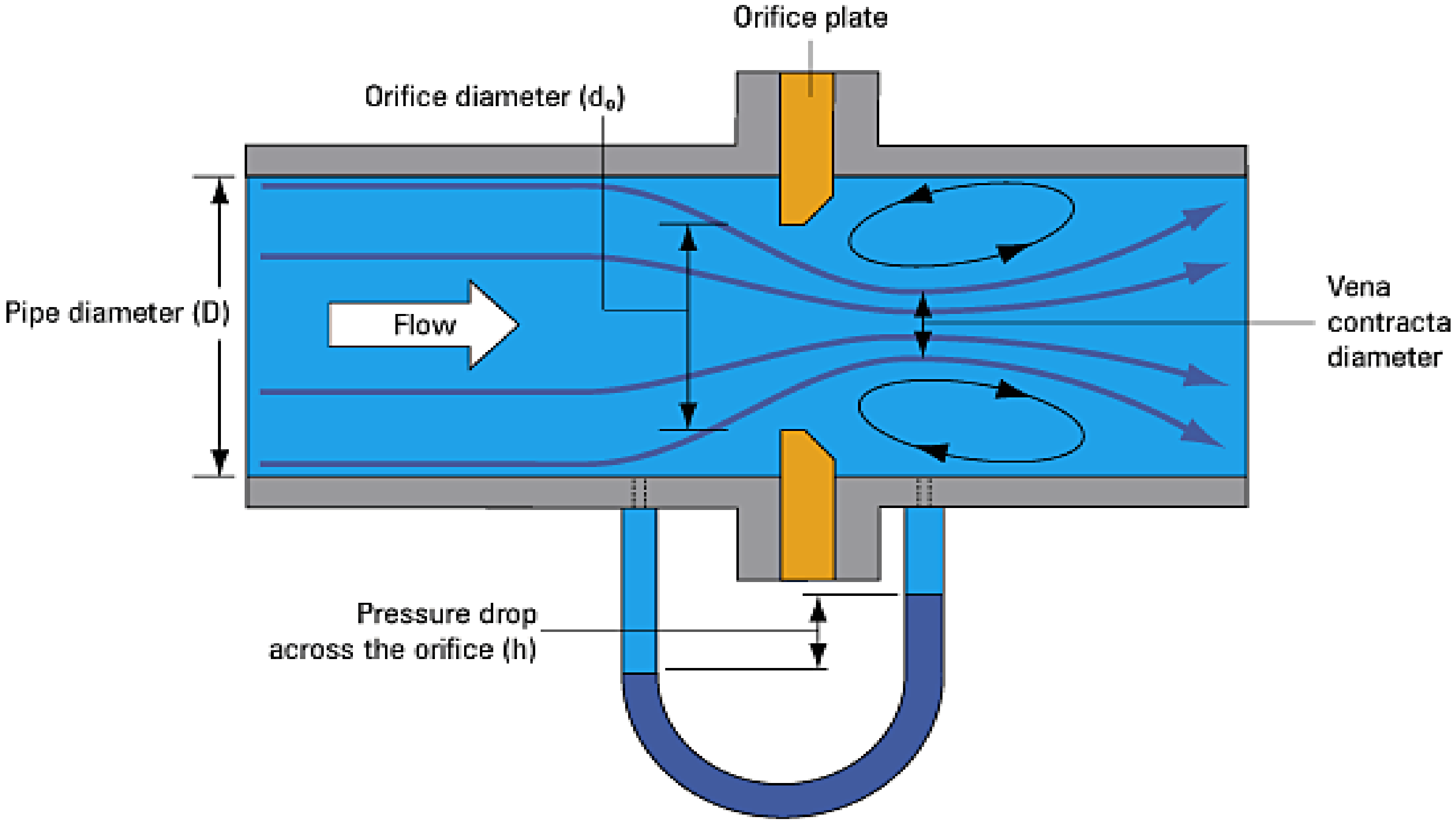
$$Q = C_d Q_{ideal} = 0.96 \times 0.85 = 0.816\ m^3/s$$

Orifice meter

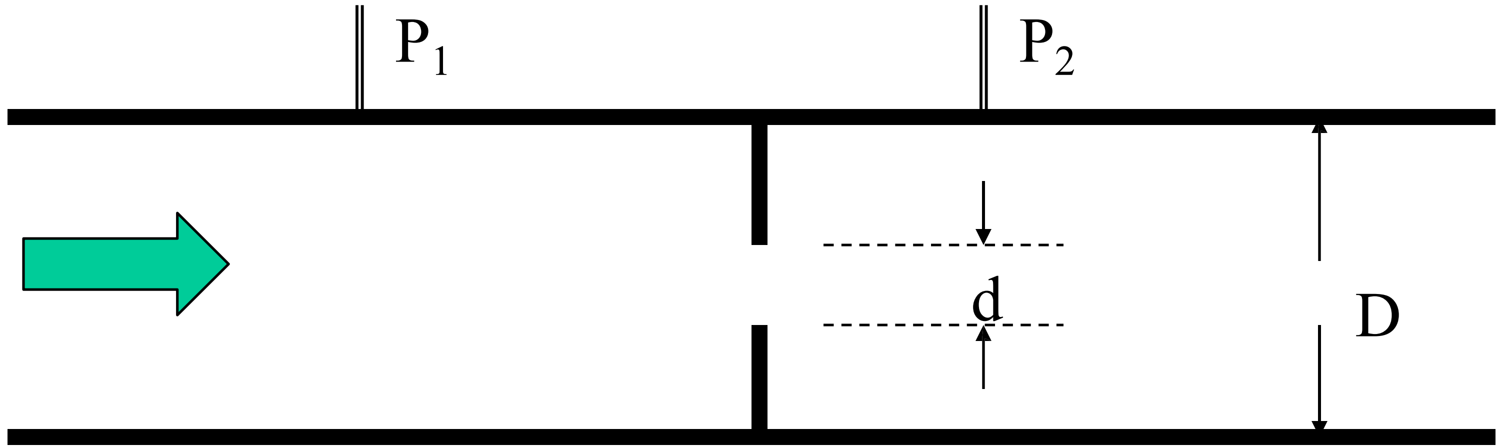
- Venturi is expensive and complex to construct.
- To measure flow rate in small pipeline → orifice meter
- Orifice meter → cheap, easy to construct

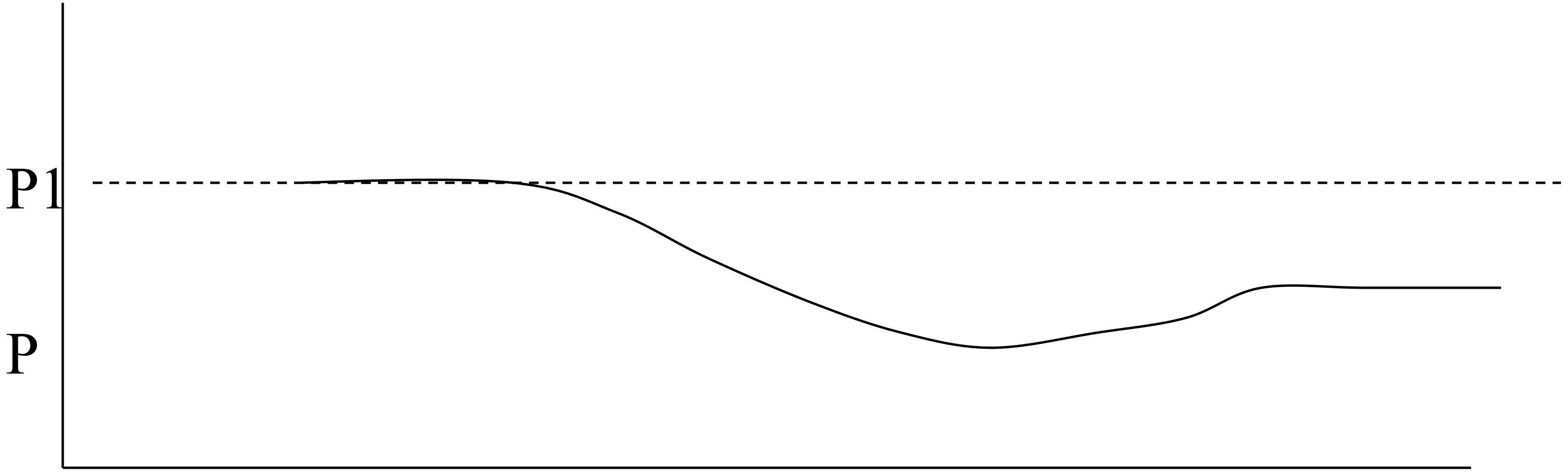


- Consist of flat orifice plate with circular hole drilled in it.
- Vena contracta → contraction of flow occurs
- Cause actual outlet velocity less than ideal outlet velocity



Graphical representation how to changes pressure with velocity of fluid flow





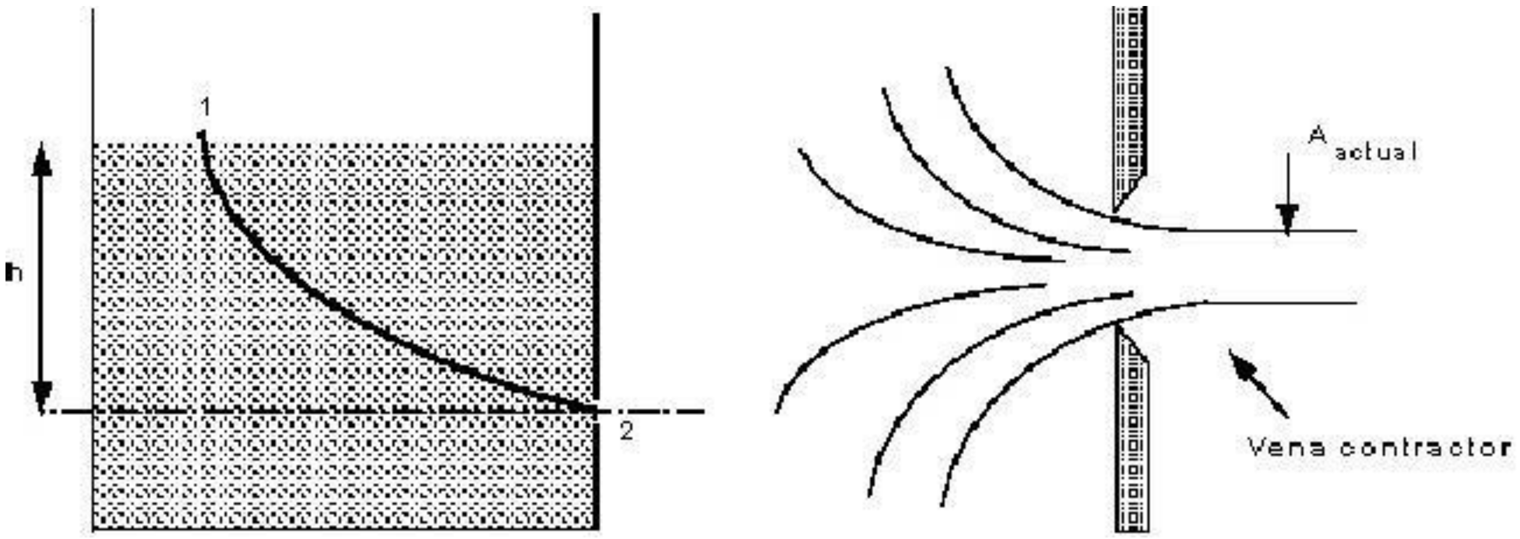


Fig. Tank and streamlines of flow out of the sharp edged orifice

We can predict the velocity of orifice using Bernounill equation.

At the surface velocity is negligible ($u_1 = 0$) and the $p_1=0$. At the orifice the jet is open to the air so $P_2=0$.

If e take the datum line through the orfice then, $z_1 = h$ and $z_2 = 0$, leaving

$$h = u_2^2/2g$$

or $$u_2 = \sqrt{2gh}$$

This is the theoretical value of velocity, and here friction losses have not taken into account. To incorporate friction, e use the co-efficient of velocity to correct the theoretical velocity,

$$u_{actual} = C_v\, u_{theoretical}$$

Each orifice has its on coefficient of velocity (0.97-0.99)

To calculate the discharge, e multiply the area of the jet by the velocity. The actual area of the jet is the area of the vena contracta not the area of the orifice. E obtain this area by using coefficient of contraction for the orifice

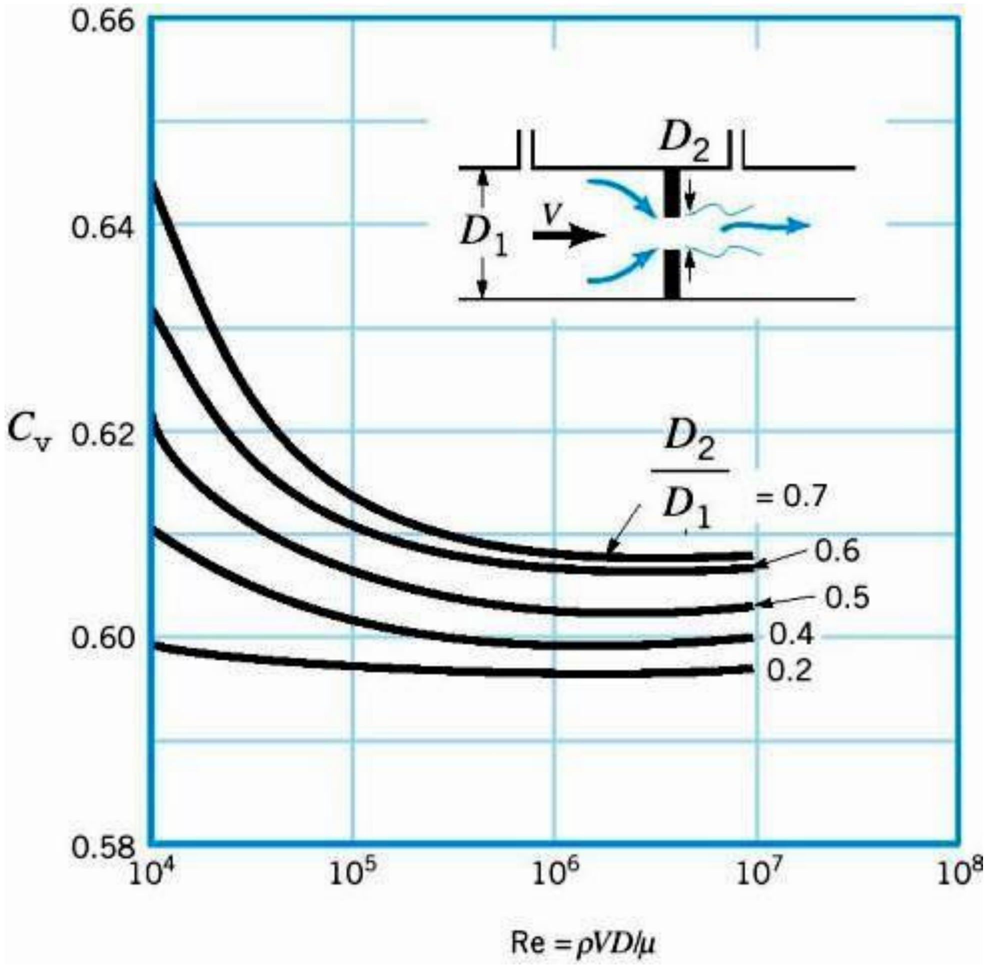
$$A_{actual} = C_c\, A_{orifice}$$

So the discharge through the orifice is given by

$$Q = Au$$

$$Q_{actual} = A_{actual}\, u_{actual}$$

$$= C_c C_v\, A_{orifice}\, u_{theoretical}$$



0.66
0.64
0.62
0.60
0.58
C_v
D_2
D_1
V
$\frac{D_2}{D_1}$ = 0.7
0.6
0.5
0.4
0.2
10^4
10^5
10^6
10^7
10^8
Re = $\rho VD/\mu$

The Momentum Equation

- In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of Newton's laws of motion.
- Account is also taken for the special properties of fluids when in motion

Newton's 2^{nd} Law can be written:

The Rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force

Derivation of momentum equation

To determine the rate of change of momentum for a fluid we will consider a stream tube as we did for the Bernoulli equation.

Assumption: steady non-uniform Flow flowing in a stream tube

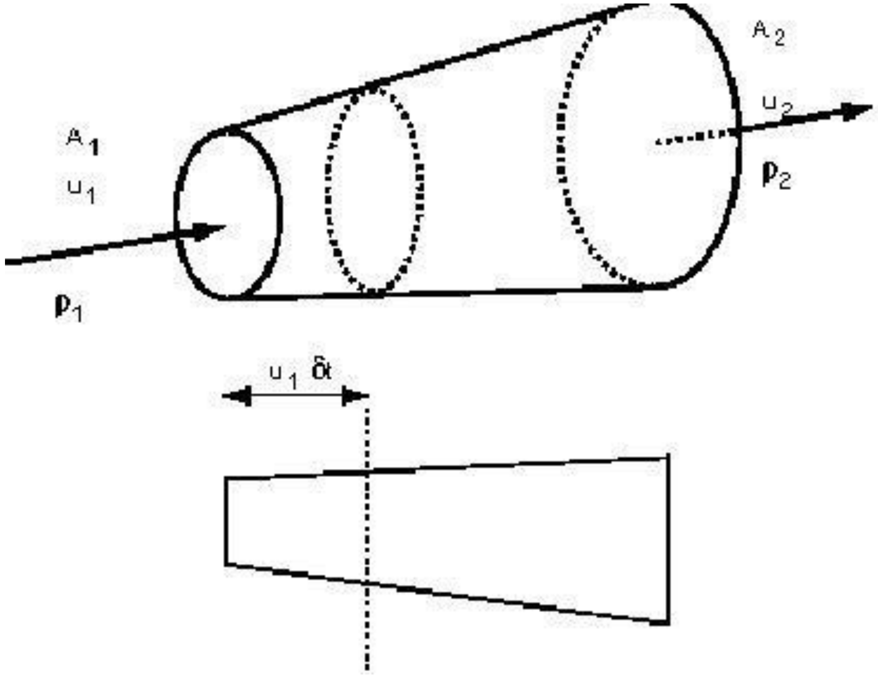


Fig. A stream tube in three and two dimensions

In time δt a volume of the fluid moves from the inlet a Distance $u\delta t$, so the volume entering the stream tube in the time δt is

Volume =Area x distance $= A_1 u_1 \delta t$

Mass $= \rho_1 A_1 u_1 \delta t$

Momentum =mass x velocity $= \rho_1 A_1 u_1 \delta t\ u_1$

Similarly, at the exit, we can obtain an expression for the momentum leaving the steam tube

Momentum of fluid leaving stream tube $= \rho_2 A_2 u_2 \delta t\ u_2$

We can now calculate the force exerted by the fluid using Newton's 2nd Law. The force is equal to the rate of change of momentum. So

Force = rate of change of momentum

$$F = \frac{\rho_2 A_2 u_2 \partial t u_2 - \rho_1 A_1 u_1 \partial t u_1}{\partial t}$$

We know from continuity that $Q=A_1u_1=A_2u_2$ and if we have a fluid of constant density, i.e. $\rho_1= \rho_2= \rho$, then we can write

$$F = Q\rho(u_2 - u_1)$$

This analysis assumed that the inlet and outlet velocities were in the same direction - i.e. a one dimensional system. What happens when this is not the case?

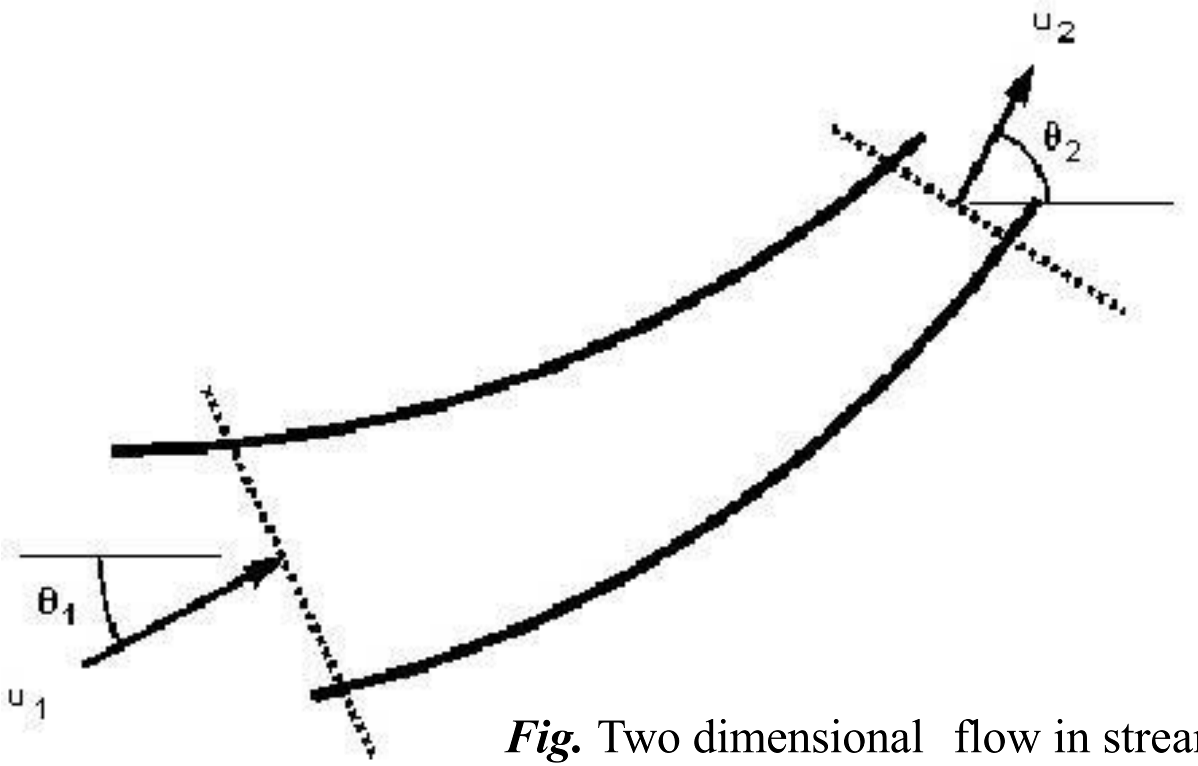


Fig. Two dimensional flow in stream tube

The force in the X-direction = rate of change of momentum in X directions

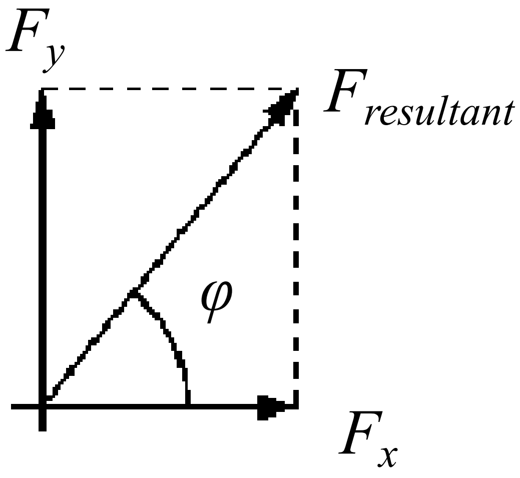
= rate of change of mass x velocity change in X direction

$$F_x = \dot{m}(u_2 \cos\theta_2 - u_1 \cos\theta_1)$$

$$F_x = Q\rho(u_2 \cos\theta_2 - u_1 \cos\theta_1)$$

The force in the y-direction $$F_y = Q\rho(u_2 \sin\theta_2 - u_1 \sin\theta_1)$$

We can find the resultant force by combining these vectorially



And the angle which this force acts at can be calculated

In summary we can say,

The total force the fluid = rate of change of momentum through the control volume

$$F = \dot{m}(u_{out} - u_{in}) = Q\rho(u_{out} - u_{in})$$

Application of the Momentum Equation

We will consider the following examples:

- Force due to the flow of fluid round a pipe bend
- Force on a nozzle at the outlet of a pipe
- Impact of a jet on a plane surface
- Force due to flow round a curved vane

Summary

- *Today emphasized basically on the application of Bernoulli equation in order to solve problems related to fluid mechanics*
- *Introduction of momentum equation in fluid mechanic*
- *Students should concentrate more on the examples and try to relate the concept in the real scenario.*

Check List For Chapter 3

- ✓ **Introduce concepts necessary to analyze fluids in motion**
- ✓ **Identify differences between Steady/unsteady uniform/non-uniform compressible/incompressible flow**
- ✓ **Introduce conservation of mass principle**
  - Continuity equation and their application in solving problems
  - Steady state mass balance
  - Unsteady state mass balance
- ✓ **Introduce Conservation of energy principle**
  - First Law of Thermodynamics
  - Derive Bernoulli's equation: Energy head and total head
  - Steady state energy balance
  - Application of Bernoulli' s equation: Fluid flow measurement
- ✓ **Introduce conservation of momentum principle**
  - Momentum transfer: the concept of balance of forces.