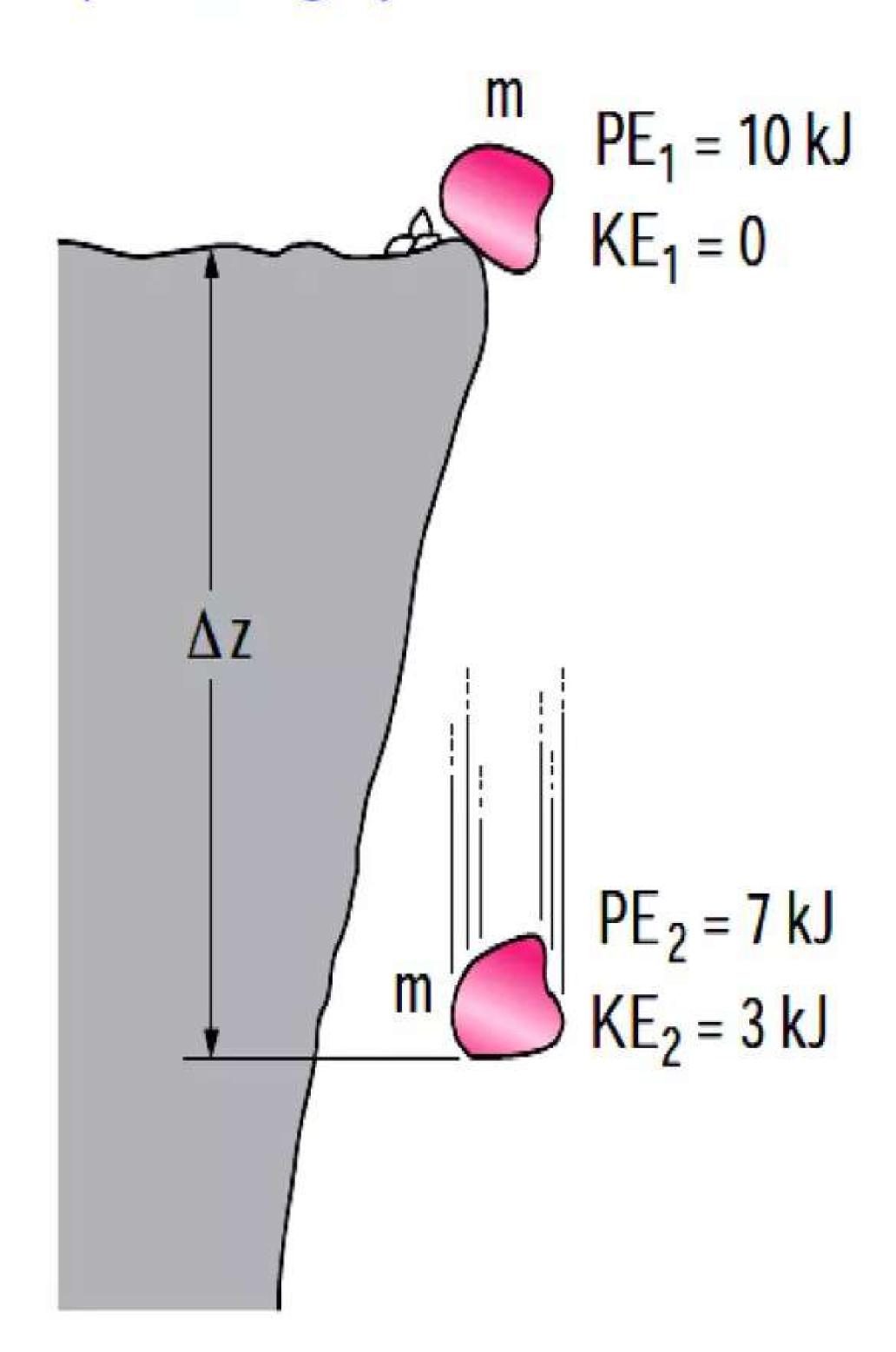
#### THE ENERGY EQUATION

• The first law of thermodynamics, also known as the conservation of energy principle, states that energy can be neither created nor destroyed during a process; it can only change forms.



Energy cannot be created or destroyed during a process; it can only change forms.

## The Energy Equation

• The change in the energy content of a system is equal to the difference between the energy input and the energy output, and the conservation of energy principle for any system can be expressed simply as

$$E_{in} - E_{out} = \Delta E$$

• The energy content of a fixed quantity of mass (a closed system) can be changed by two mechanisms: *heat transfer Q* and *work transfer W*. Then the conservation of energy for a fixed quantity of mass can be expressed in rate form as

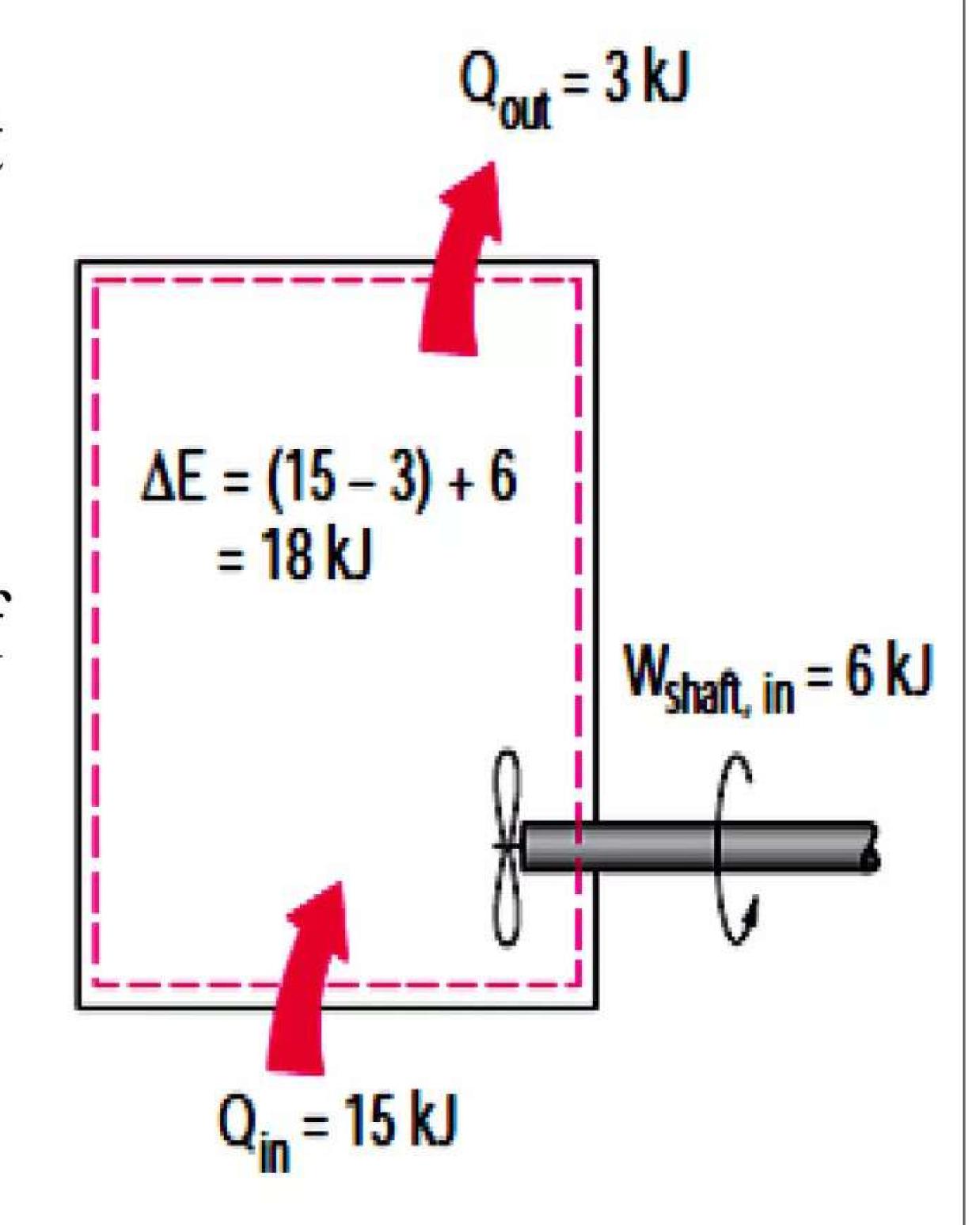
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{\text{sys}}}{dt}$$
 or  $\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt} \int_{\text{sys}} \rho e \, dV$ 

• Where  $\dot{Q}_{net in} = \dot{Q}_{in} - \dot{Q}_{out}$  is the net rate of heat transfer to the system (negative, if from the system)

## The Energy Equation

- $W_{\text{net in}} = W_{\text{in}} W_{\text{out}}$  is the net power input to the system in all forms (negative, if power output)
- dE<sub>sys</sub>/dt is the rate of change of the total energy content of the system.

(N.B: The overdot stands for time rate)



For simple compressible systems, total energy consists of internal, kinetic, and potential energies, and it is expressed on a unit-mass basis as

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$

## Energy Transfer by Heat, Q

- The transfer of thermal energy from one system to another as a result of a temperature difference is called **heat transfer**.
- A process during which there is no heat transfer is called an adiabatic process.
- There are two ways a process can be adiabatic: Either the system is well insulated so that only a negligible amount of heat can pass through the system boundary, or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer.
- An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work transfer.

## Energy Transfer by Work, W

- An energy interaction is work if it is associated with a force acting through a distance.
- A rising piston, a rotating shaft, and an electric wire crossing the system boundary are all associated with work interactions.
- The time rate of doing work is called **power** and is denoted by W.
- Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.
- Work-consuming devices transfer energy to the fluid, and thus increase the energy of the fluid. A fan in a room, for example, mobilizes the air and increases its kinetic energy.

## Energy Transfer by Work, W

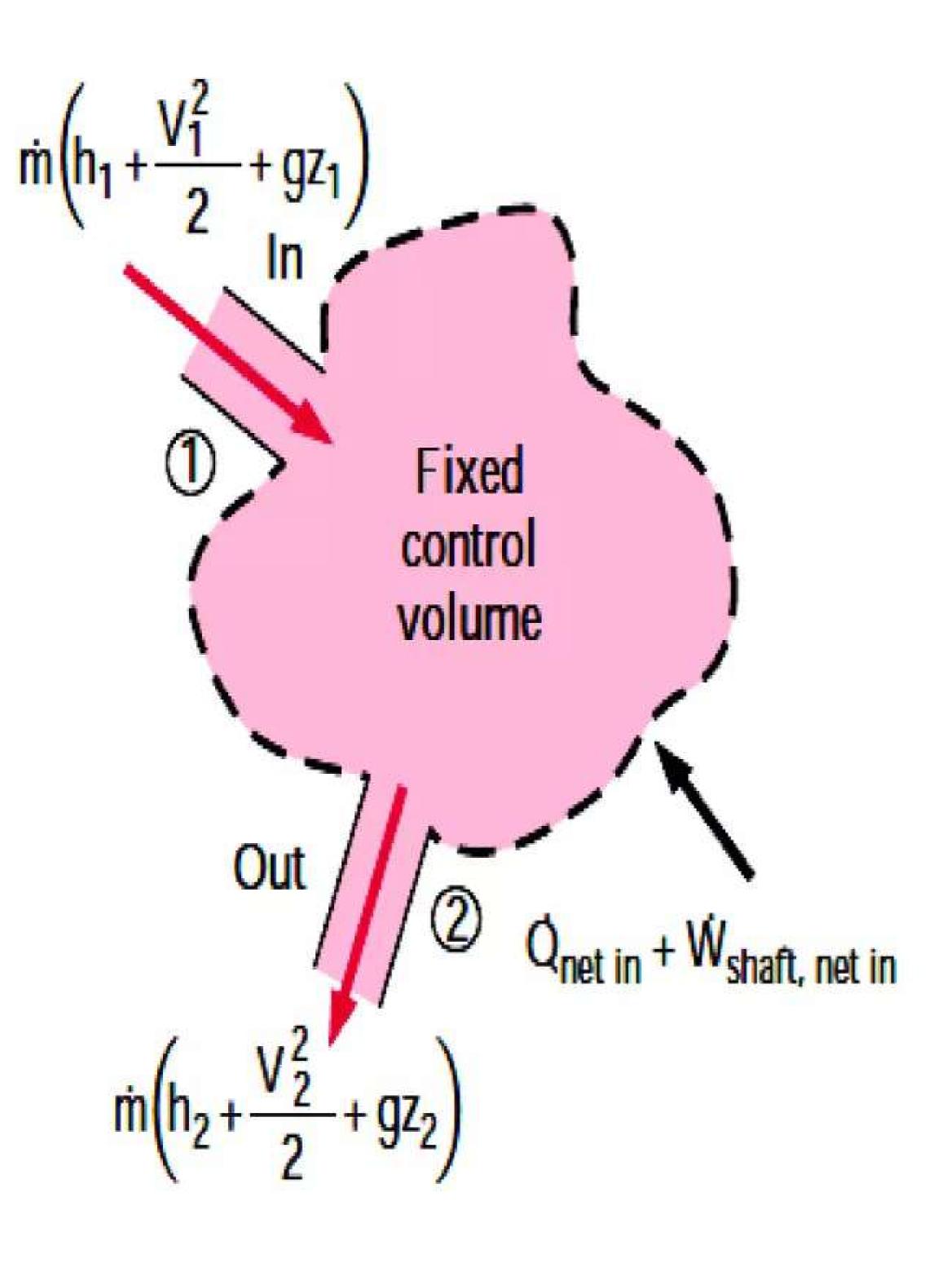
• A system may involve numerous forms of work, and the total work can be expressed as

- W<sub>shaft</sub> is the work transmitted by a rotating shaft
- W<sub>pressure</sub> is the work done by the pressure forces on the control surface,
- W<sub>viscous</sub> is the work done by the normal and shear components of viscous forces on the control surface,
- W<sub>other</sub> is the work done by other forces such as electric, magnetic, and surface tension, which are insignificant for simple compressible systems
- ullet  $W_{viscous}$  is usually very small relative to other terms. So it is not considered in control volume analysis.

• For steady flows, the energy equation is given by

$$\dot{Q}_{net in} + \dot{W}_{shaft, net in} = \sum_{out} \dot{m} \left( h + \frac{V^2}{2} + gz \right) - \sum_{in} \dot{m} \left( h + \frac{V^2}{2} + gz \right)$$

• It states that the net rate of energy transfer to a control volume by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.



• Many practical problems involve just one inlet and one outlet. The mass flow rate for such **single-stream devices** remains constant, and the energy equation reduces to

$$\dot{Q}_{net in} + \dot{W}_{shaft, net in} = \dot{m} \left( h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right)$$

- where subscripts 1 and 2 stand for inlet and outlet, respectively.
- on a unit-mass basis

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

• Using the definition of enthalpy  $h = u + P/\rho$  and rearranging, the steady-flow energy equation can also be expressed as

$$W_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$

• where u is the internal energy, P/ $\rho$  is the flow energy, V<sup>2</sup>/2 is the kinetic energy, and gz is the potential energy of the fluid, all per unit mass. These relations are valid for both compressible and incompressible flows

$$W_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$

$$Mechanical \ Energy$$

$$Input$$

$$Output$$

• If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved. Thus  $u_2 - u_1 - q_{\text{net in}}$  must be equal to zero.

Ideal flow (no mechanical energy loss):

 $q_{\text{net in}} = u_2 - u_1$ 

•  $u_2$  -  $u_1$  -  $q_{net in}$  represents the mechanical energy loss

Mechanical energy loss:

$$e_{mech, loss} = u_2 - u_1 - q_{net in}$$

• For single-phase fluids (a gas or a liquid), we have

$$u_2 - u_1 = c_v(T_2 - T_1)$$

where c<sub>v</sub> is the constant-volume specific heat.

• The steady-flow energy equation on a unit-mass basis can be written conveniently as a mechanical energy balance as

$$W_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}}$$

Noting that

• the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{pump} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{turbine} + e_{mech, loss}$$

- where w<sub>pump</sub> is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and w<sub>turbine</sub> is the mechanical work output.
- Multiplying the above energy equation by the mass flow rate **m** gives

$$\dot{m} \left( \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{pump} = \dot{m} \left( \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{turbine} + \dot{E}_{mech, loss}$$

#### Where

- Whomp is the shaft power input through the pump's shaft,
- Wurbine turbine is the shaft power output through the turbine's shaft, and
- E<sub>mech, loss</sub>, loss is the *total* mechanical power loss, which consists of pump and turbine losses as well as the frictional losses in the piping network.

$$\dot{E}_{mech, loss} = \dot{E}_{mech loss, pump} + \dot{E}_{mech loss, turbine} + \dot{E}_{mech loss, piping}$$

• The energy equation can be expressed in its most common form in terms of heads as

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump, u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine, e} + h_L$$

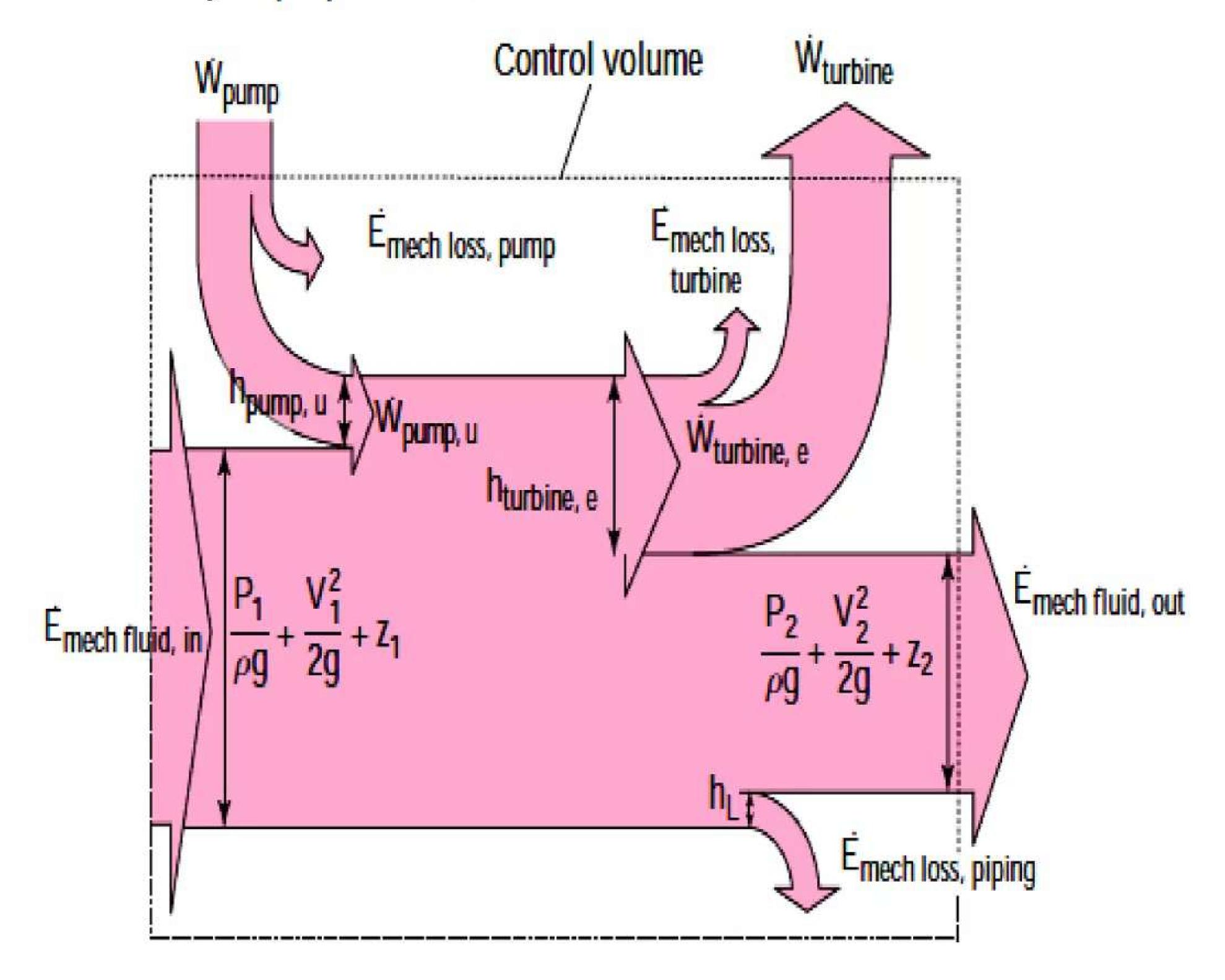
Where

$$h_{pump, u} = \frac{W_{pump, u}}{g} = \frac{\dot{W}_{pump, u}}{\dot{m}g} = \frac{\eta_{pump}\dot{W}_{pump}}{\dot{m}g}$$
 is the useful head delivered to the fluid by the pump

$$h_{turbine, e} = \frac{W_{turbine, e}}{g} = \frac{W_{turbine, e}}{\dot{m}g} = \frac{\dot{W}_{turbine}}{\eta_{turbine}\dot{m}g}$$
 is the extracted head removed from the fluid by the turbine.

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{E_{\text{mech loss, piping}}}{\dot{m}g} \quad \text{is the irreversible head}$$
 loss between 1 and 2 due to all components of the piping system other than the pump or turbine.

• Note that the head loss  $h_L$  represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices—these losses are taken into account by  $\eta_{\text{pump}}$  and  $\eta_{\text{turbine}}$ 



- The pump head is zero if the piping system does not involve a pump, a fan, or a compressor, and the turbine head is zero if the system does not involve a turbine.
- Also, the head loss h<sub>L</sub> can sometimes be ignored when the frictional losses in the piping system are negligibly small compared to the other terms

# Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

• When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus  $h_L = e_{mech\ loss,\ piping}/g \cong 0$ , and  $h_{pump,\ u} = h_{turbine,\ e} = 0$ 

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$
 or  $\frac{P}{\rho g} + \frac{V^2}{2g} + z = constant$ 

which is the Bernoulli equation

#### Kinetic Energy Correction Factor, a

- the kinetic energy of a fluid stream obtained from V<sup>2</sup>/2 is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components
- This error can be corrected by replacing the kinetic energy terms  $V^2/2$  in the energy equation by  $\alpha V_{avg}^2/2$ , where  $\alpha$  is the kinetic energy correction factor.
- The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.
- The kinetic energy correction factors are often ignored (i.e., a is set equal to 1) in an elementary analysis since (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and

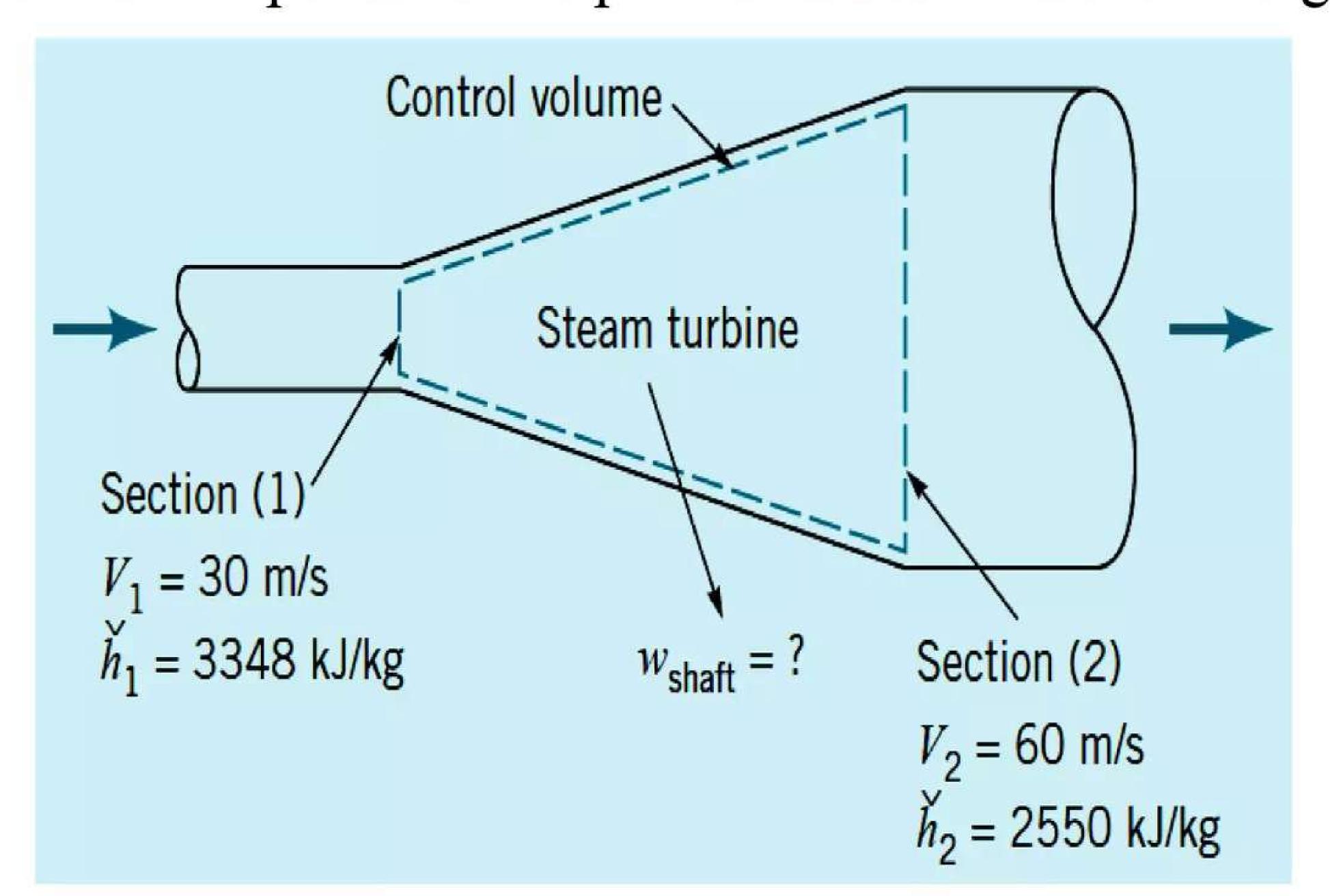
- (2) the kinetic energy terms are often small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference.
- When the kinetic energy correction factors are included, the energy equations for *steady incompressible flow* become

$$\dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right) + W_{pump} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) + W_{turbine} + \dot{E}_{mech, loss}$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine, e} + h_L$$

#### Example 1

• A steam turbine generator unit used to produce electricity. Assume the steam enters a turbine with a velocity of 30 m/s and enthalpy, h<sub>1</sub>, of 3348 kJ/kg. The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. The flow through the turbine is adiabatic, and changes in elevation are negligible. Determine the work output involved per unit mass of steam through-flow.



#### Solution

0 (elevation change is negligible)

$$\dot{m} \left[ \dot{h}_2 - \ddot{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}}$$
 (1)

The work output per unit mass of steam through-flow,  $w_{\text{shaft net in}}$ , can be obtained by dividing Eq. 1 by the mass flow rate,  $\dot{m}$ , to obtain

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}} = \dot{h}_2 - \dot{h}_1 + \frac{V_2^2 - V_1^2}{2} \tag{2}$$

Since  $w_{\text{shaft net out}} = -w_{\text{shaft net in}}$ , we obtain

$$w_{\text{shaft net out}} = \check{h}_1 - \check{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

#### Solution

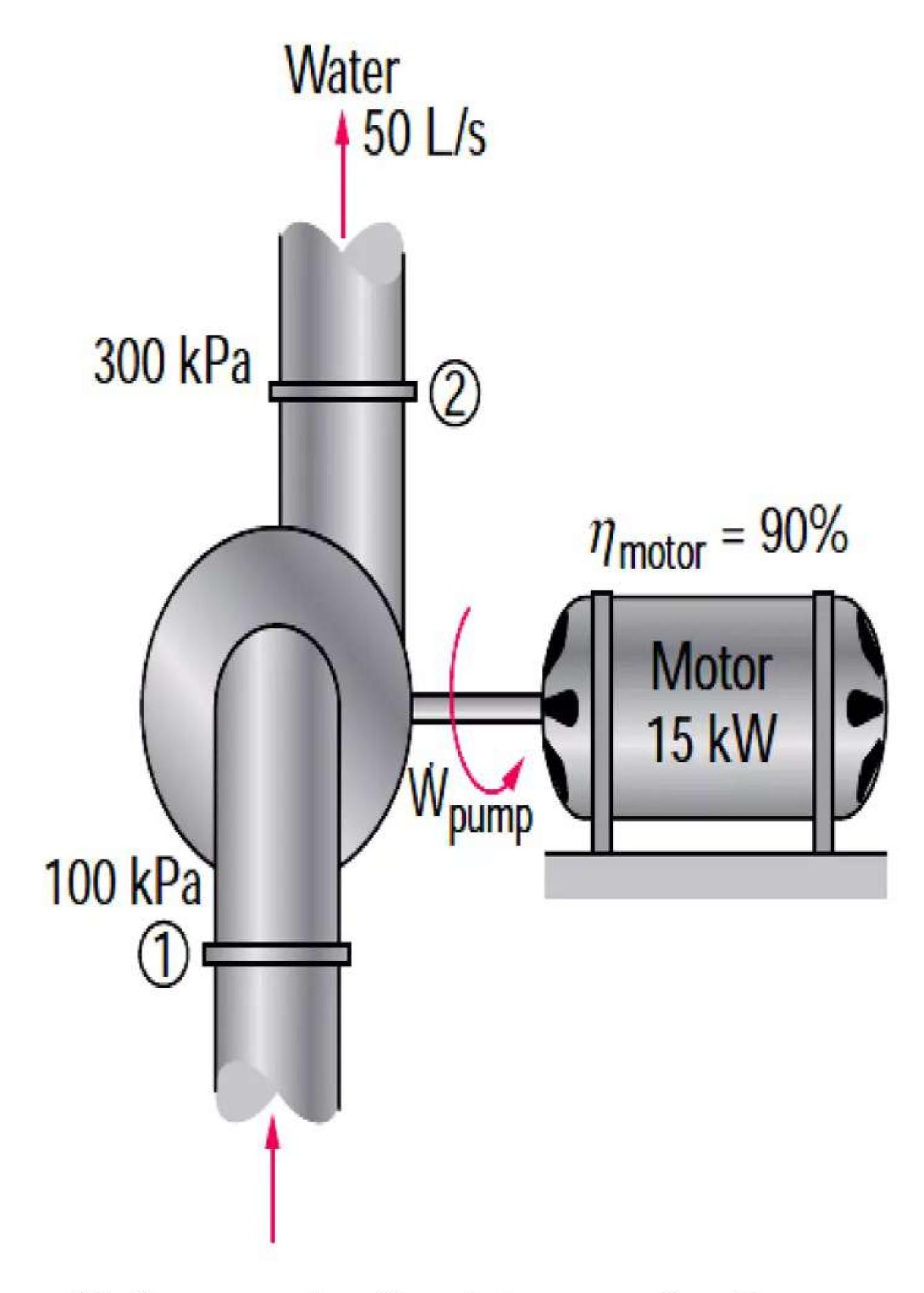
$$w_{\text{shaft}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg}$$

$$+ \frac{[(30 \text{ m/s})^2 - (60 \text{ m/s})^2][1 \text{ J/(N·m)}]}{2[1 \text{ (kg·m)/(N·s}^2)](1000 \text{ J/kJ})}$$

Thus,

$$w_{\text{shaft net out}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg} - 1.35 \text{ kJ/kg}$$
  
= 797 kJ/kg (Ans)

- The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent. The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine
  - a) the mechanical efficiency of the pump and
  - b) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.



Schematic for Example 2

Assumptions 1 The flow is steady and incompressible. 2 The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. 3 The elevation difference between the inlet and outlet of the pump is negligible,  $z_1 \cong z_2$ . 4 The inlet and outlet diameters are the same and thus the inlet and outlet velocities and kinetic energy correction factors are equal,  $V_1 = V_2$  and  $\alpha_1 = \alpha_2$ .

**Properties** We take the density of water to be  $1 \text{ kg/L} = 1000 \text{ kg/m}^3$  and its specific heat to be  $4.18 \text{ kJ/kg} \cdot ^{\circ}\text{C}$ .

Analysis (a) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5 \text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) - \dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m} \left( \frac{P_2 - P_1}{\rho} \right) = (50 \text{ kg/s}) \left( \frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3} \right) \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = 10 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10 \text{ kW}}{13.5 \text{ kW}} = 0.741 \text{ or } 74.1\%$$

(b) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5 \text{ kW}$$

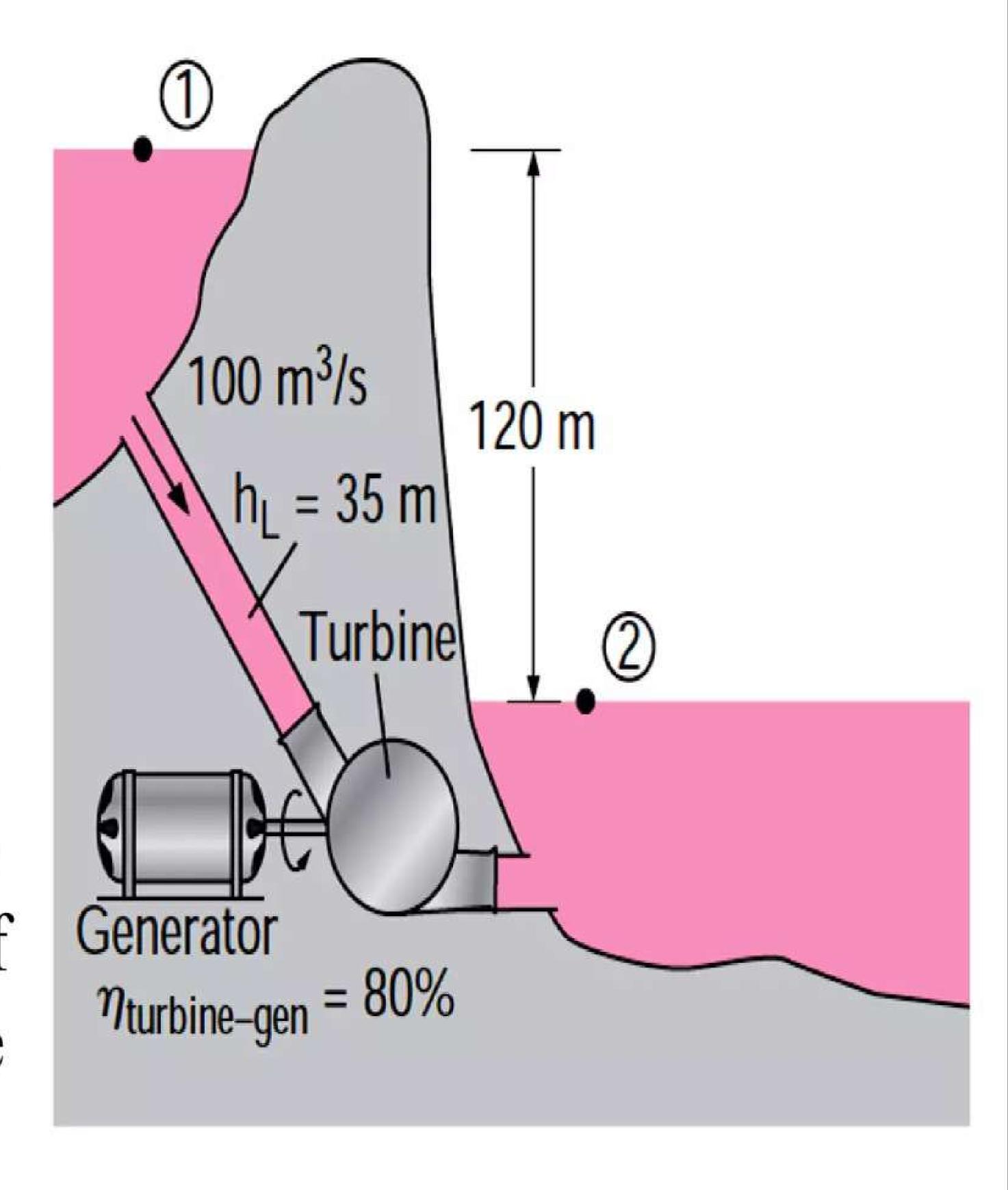
The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance,  $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$ . Solving for  $\Delta T$ ,

$$\Delta T = \frac{\dot{E}_{mech, loss}}{\dot{m}c} = \frac{3.5 \text{ kW}}{(50 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^{\circ}\text{C})} = 0.017^{\circ}\text{C}$$

Therefore, the water will experience a temperature rise of 0.017°C due to mechanical inefficiency, which is very small, as it flows through the pump.

#### **Example 3. Hydroelectric Power Generation from a Dam**

• In a hydroelectric power plant, 100 m<sup>3</sup>/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine-generator is 80 percent, estimate the electric power output.



#### **Example 3. Hydroelectric Power Generation from a Dam**

**SOLUTION** The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined. **Assumptions** 1 The flow is steady and incompressible. 2 Water levels at the reservoir and the discharge site remain constant.

**Properties** We take the density of water to be 1000 kg/m<sup>3</sup>.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho \dot{V} = (10000 \text{ kg/m}^3)(100 \text{ m}^3/\text{s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus  $z_2 = 0$ . Also, both points 1 and 2 are open to the atmosphere ( $P_1 = P_2 = P_{atm}$ ) and the flow velocities are negligible at both points ( $V_1 = V_2 = 0$ ). Then the energy equation for steady, incompressible flow reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{\sqrt{2}}{2g} + z_1 + h_{pump, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{\sqrt{2}}{2g} + z_2 + h_{turbine, e} + h_L \rightarrow h_{turbine, e} = z_1 - h_L$$

#### **Example 3. Hydroelectric Power Generation from a Dam**

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine, e}} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$
/ 1 kJ/kg \

$$\dot{W}_{\text{turbine, e}} = \dot{m}gh_{\text{turbine, e}} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m}) \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 83,400 \text{ kW}$$

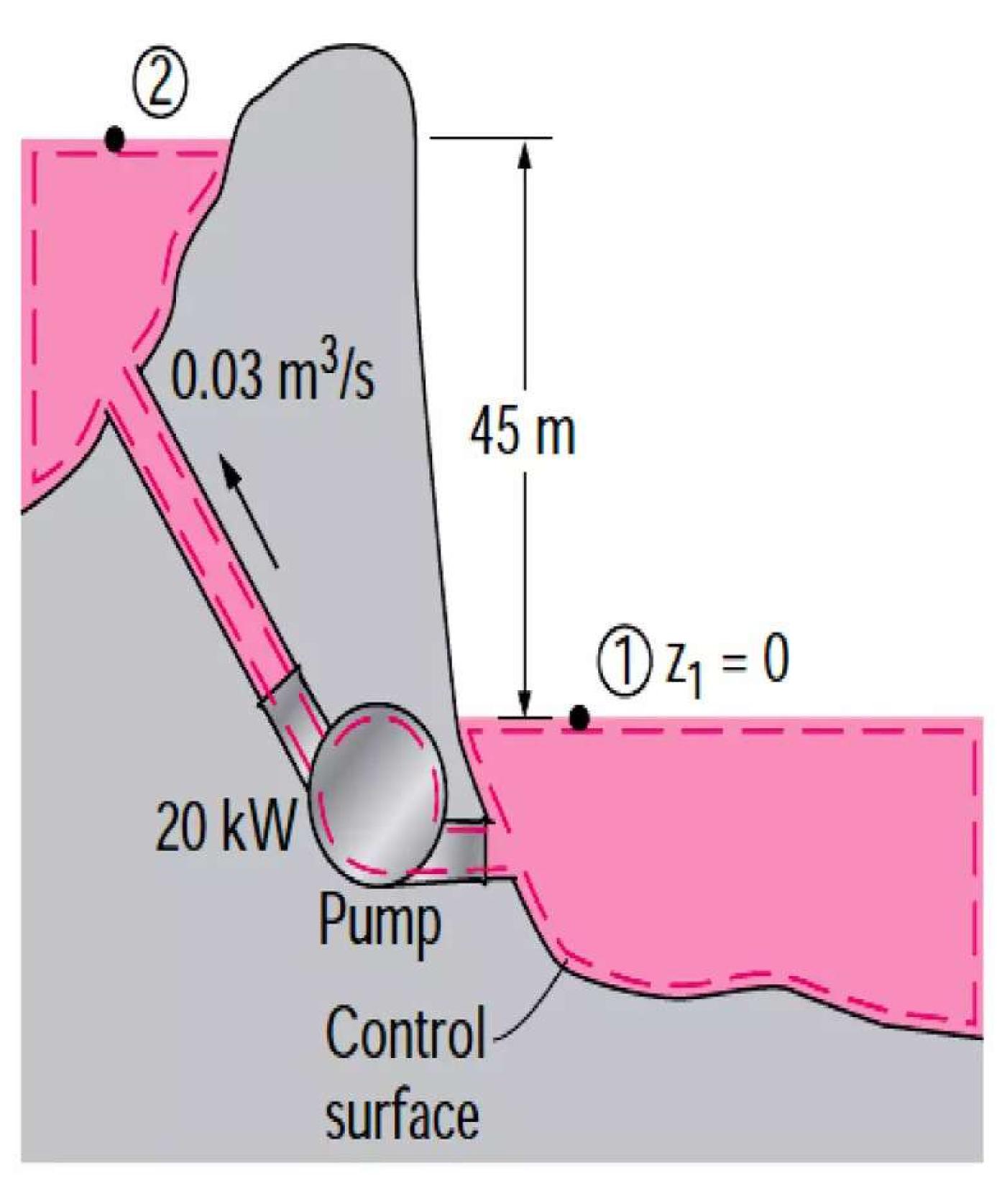
Therefore, a perfect turbine-generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{electric} = \eta_{turbine-gen} \dot{W}_{turbine, e} = (0.80)(83.4 \text{ MW}) = 66.7 \text{ MW}$$

**Discussion** Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit.

#### **Example 4. Head and Power Loss During Water Pumping**

• Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m<sup>3</sup>/s, determine the irreversible head loss of the system and the lost mechanical power during this process.



**Properties** We take the density of water to be 1000 kg/m<sup>3</sup>. **Analysis** The mass flow rate of water through the system is

$$\dot{m} = \rho \dot{V} = (10000 \text{ kg/m}^3)(0.03 \text{ m}^3/\text{s}) = 30 \text{ kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ( $z_1 = 0$ ). Both points are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ) and the velocities at both locations are negligible ( $V_1 = V_2 = 0$ ). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\dot{m} \left( \frac{P_1}{P} + \alpha_1 \frac{V_1^2}{2} + gz_1^{-0} \right) + \dot{W}_{pump}$$

$$= \dot{m} \left( \frac{P_2}{P} + \alpha_2 \frac{V_2^2}{2} \right) + \dot{W}_{turbine} + \dot{E}_{mech, loss}$$

$$\dot{W}_{pump} = \dot{m}gz_2 + \dot{E}_{mech, loss} \rightarrow \dot{E}_{mech, loss} = \dot{W}_{pump} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

$$\dot{E}_{\text{mech, loss}} = 20 \text{ kW} - (30 \text{ kg/s})(9.81 \text{ m/s}^2)(45 \text{ m}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) \left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}}\right)$$

#### Example 4. Solution....

Noting that the entire mechanical losses are due to frictional losses in piping and thus  $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech, loss, piping}}$ , the irreversible head loss is determined to be

$$h_{L} = \frac{\dot{E}_{mech loss, piping}}{\dot{m}g} = \frac{6.76 \text{ kW}}{(30 \text{ kg/s})(9.81 \text{ m/s}^2)} \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) \left(\frac{1000 \text{ N} \cdot \text{m/s}}{1 \text{ kW}}\right) = \frac{23.0 \text{ m}}{1 \text{ kW}}$$

**Discussion** The 6.76 kW of power is used to overcome the friction in the piping system. Note that the pump could raise the water an additional 23 m if there were no irreversible head losses in the system. In this ideal case, the pump would function as a turbine when the water is allowed to flow from the upper reservoir to the lower reservoir and extract 20 kW of power from the water.

## End of Chapter 3

#### Next Lecture

Chapter 4: Differential Relations For A
Fluid Flow