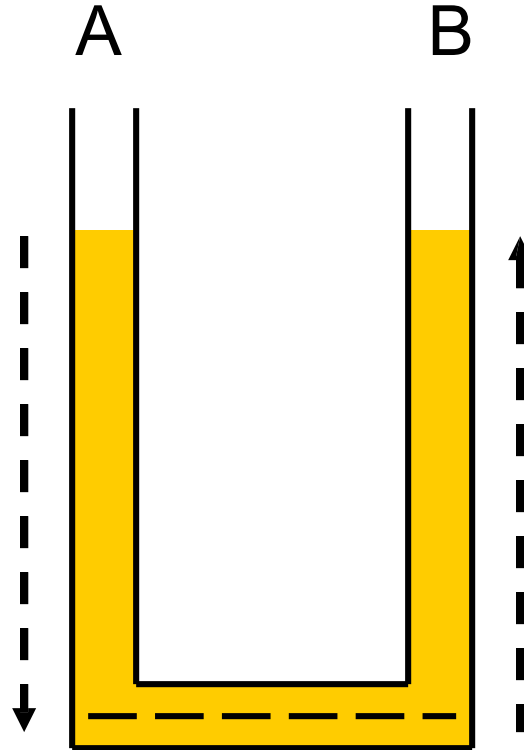


Pressure Measurement

- Objective:
 - Understand the principles of manometer
 - Learn how to calculate pressure using manometer

In measuring pressure:

- Pressure increases as one goes downward
- In calculation, has to "plus" the pressure



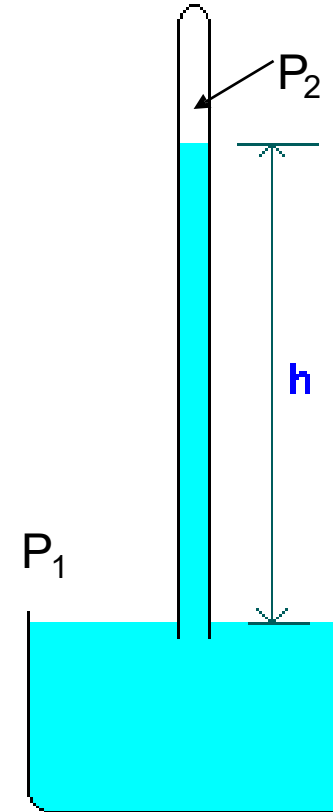
- Pressure decreases as one goes upward
- In calculation, has to "minus" the pressure

Measurement of Pressure

Barometer : (Mercury Barometer)

A barometer is used to measure atmospheric pressure

- A simple barometer consists of:
 - a tube more than 30 inch (760 mm) long
 - inserted in an open container of mercury
 - a closed and evacuated end pointing upward
 - open end in the mercury pool
 - mercury extending from the container up into the tube.
 - It contains mercury vapor at its saturated vapor pressure



Measurement of atmospheric pressure

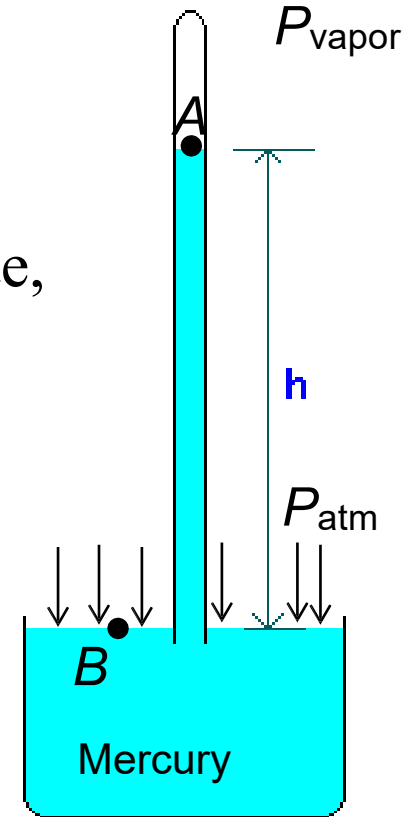
In reference to barometric equation, we can write,

$$P_{\text{atm}} = \gamma_{\text{Hg}} h + P_{\text{vapor}}$$

$$P_{\text{atm}} = \rho g h + P_{\text{vapor}}$$

Since pressure exerted by mercury vapor is very small, therefore,

$$P_{\text{atm}} = \gamma_{\text{Hg}} h$$



Manometer: They use vertical or inclined liquid columns to measure pressure

A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes.

- Piezometer tube
- U-tube manometer
- Inverted U-tube manometer
- Inclined tube manometer

Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured.

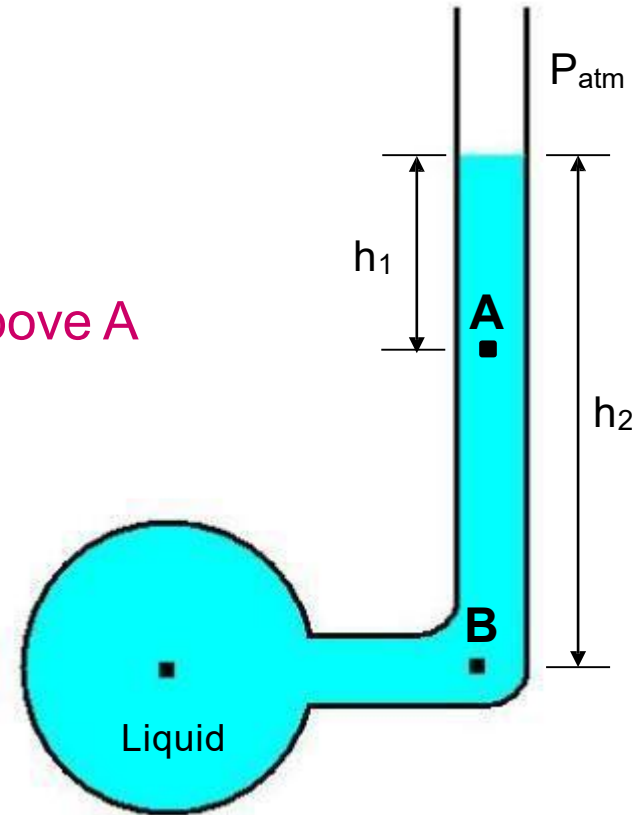
As the tube is open to the atmosphere the pressure is relative to atmospheric so is **gauge pressure**

Pressure at A = pressure due to column of liquid above A

$$P_A = \cancel{P_{atm}}^0 + \rho g h_1 = \rho g h_1$$

Pressure at B = pressure due to column of liquid above B

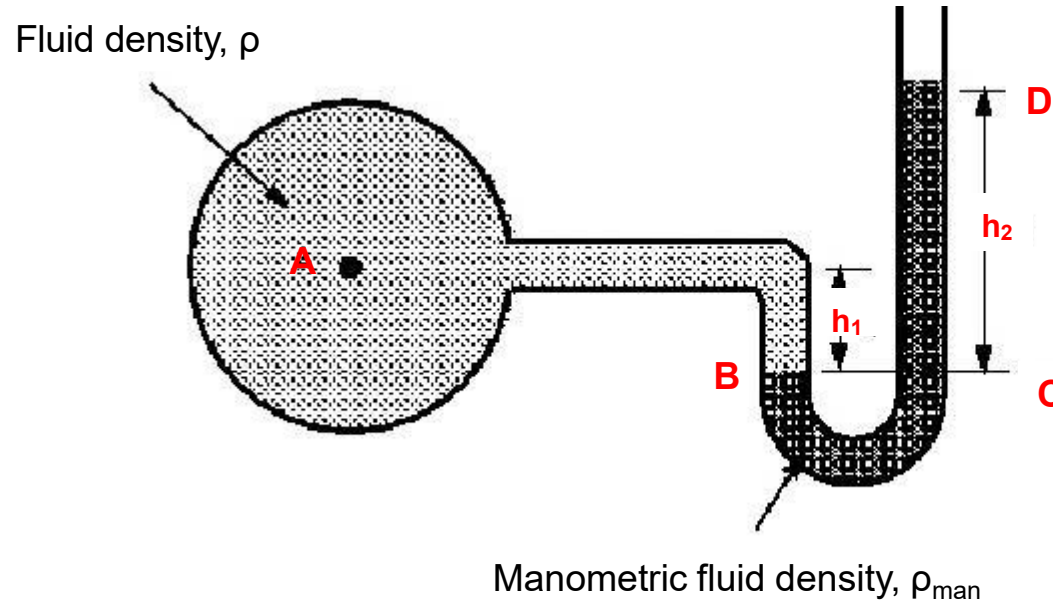
$$P_B = P_{atm} + \rho g h_2 = 0 + \rho g h_2$$



This method can only be used **for liquids** (*i.e.*, **not** for gases) and only when the liquid **height is convenient** to measure. It must not be too small or too large and pressure changes must be detectable

The “U”-Tube Manometer

Using a “U”-Tube enables the pressure of **both liquids and gases** to be measured with the same instrument. The “U” is connected as in the figure below and filled with a fluid called the manometric fluid. The fluid whose pressure is being measured should have a mass density **less than that of the manometric** fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



Pressure in a continuous static fluid is the **same at any horizontal level**
so, $P_B = P_C \dots\dots\dots(1)$

For the **left hand** arm,

Pressure at B = pressure at A + pressure due to height h_1 of measured fluid

$$P_B = P_A + \rho g h_1$$

For the **right hand** arm

Pressure at C = pressure at D + pressure due to height h of manometric fluid

$$\begin{aligned} P_C &= P_{\text{atm}} + \rho_{\text{man}} g h_2 \\ &= \rho_{\text{man}} g h_2 \quad (\text{Since we are measuring gage pressure, } P_{\text{atm}} = 0) \end{aligned}$$

Putting the values of P_B and P_C in equation 1, we find, $P_A = \rho_{\text{man}} g h_2 - \rho g h_1$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid *i.e.*, $\rho_{\text{man}} \gg \rho$. In this case the term $\rho g h_1$ can be neglected, and the gauge pressure give by

$$P_A = \rho_{\text{man}} g h_2$$

Measurement of Pressure Difference Using a “U”-Tube Manometer

If the “U”-tube manometer is connected to a pressurized vessel at two points the pressure difference between these two points can be measured.

If the manometer is arranged as in the figure, then we say

Pressure at C = Pressure at D

$$P_C = P_D \text{(1)}$$

$$P_C = P_A + \rho g h_a \text{ (2)}$$

$$P_D = P_B + \rho g (h_b - h) + \rho_{\text{man}} g h \text{(3)}$$

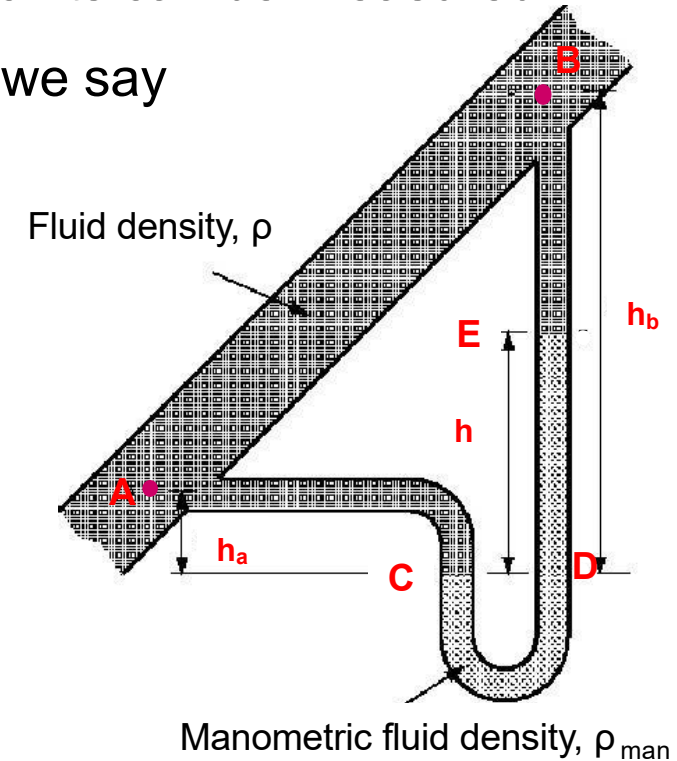
Combining equation 1, 2 & 3,

$$P_A + \rho g h_a = P_B + \rho g (h_b - h) + \rho_{\text{man}} g h$$

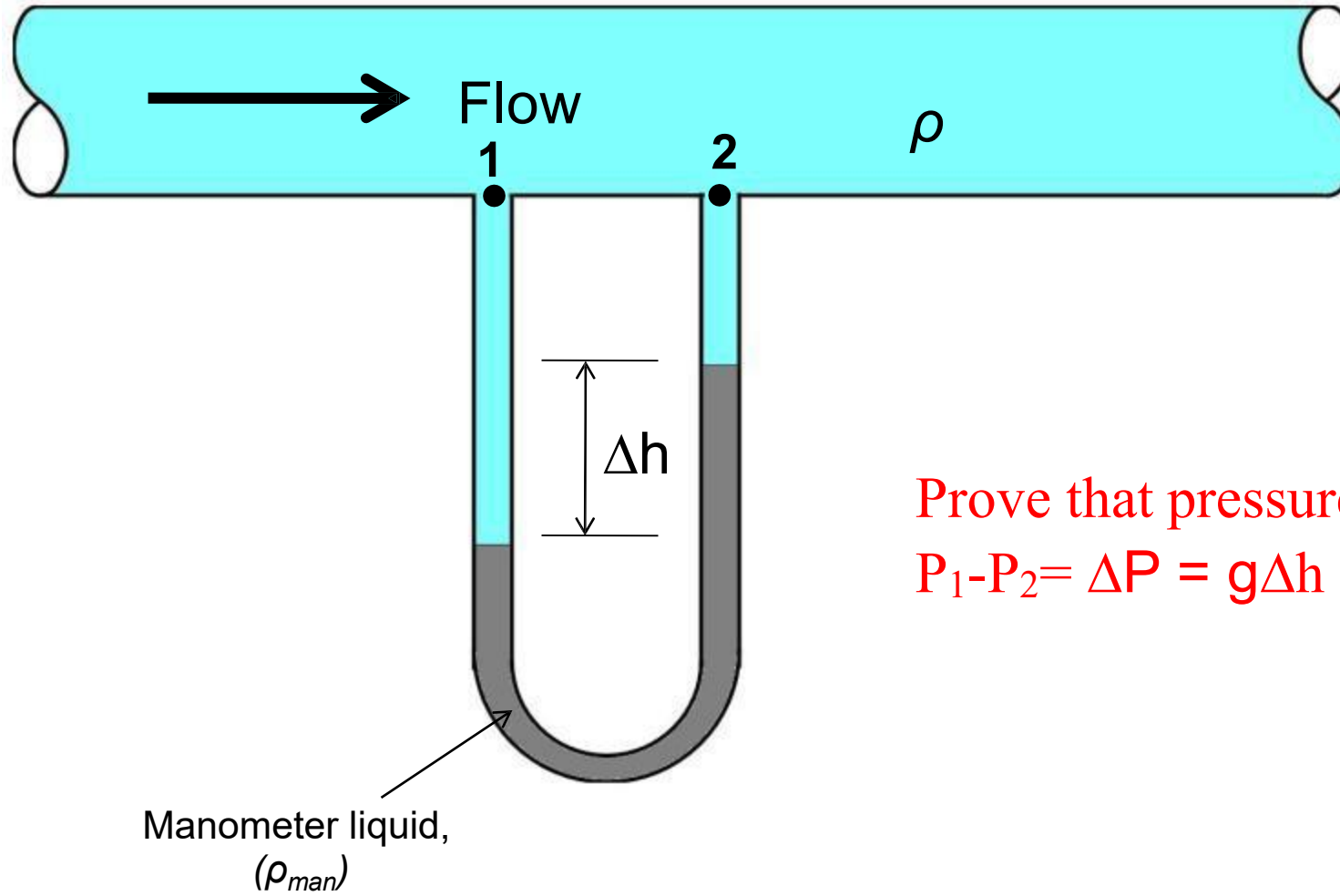
$$\text{or, } P_A - P_B = \rho g (h_b - h_a) + (\rho_{\text{man}} - \rho) g h$$

if the fluid whose pressure difference is being measured is a gas and $\rho_{\text{man}} \gg \rho$, then the terms involving ρ can be neglected, so

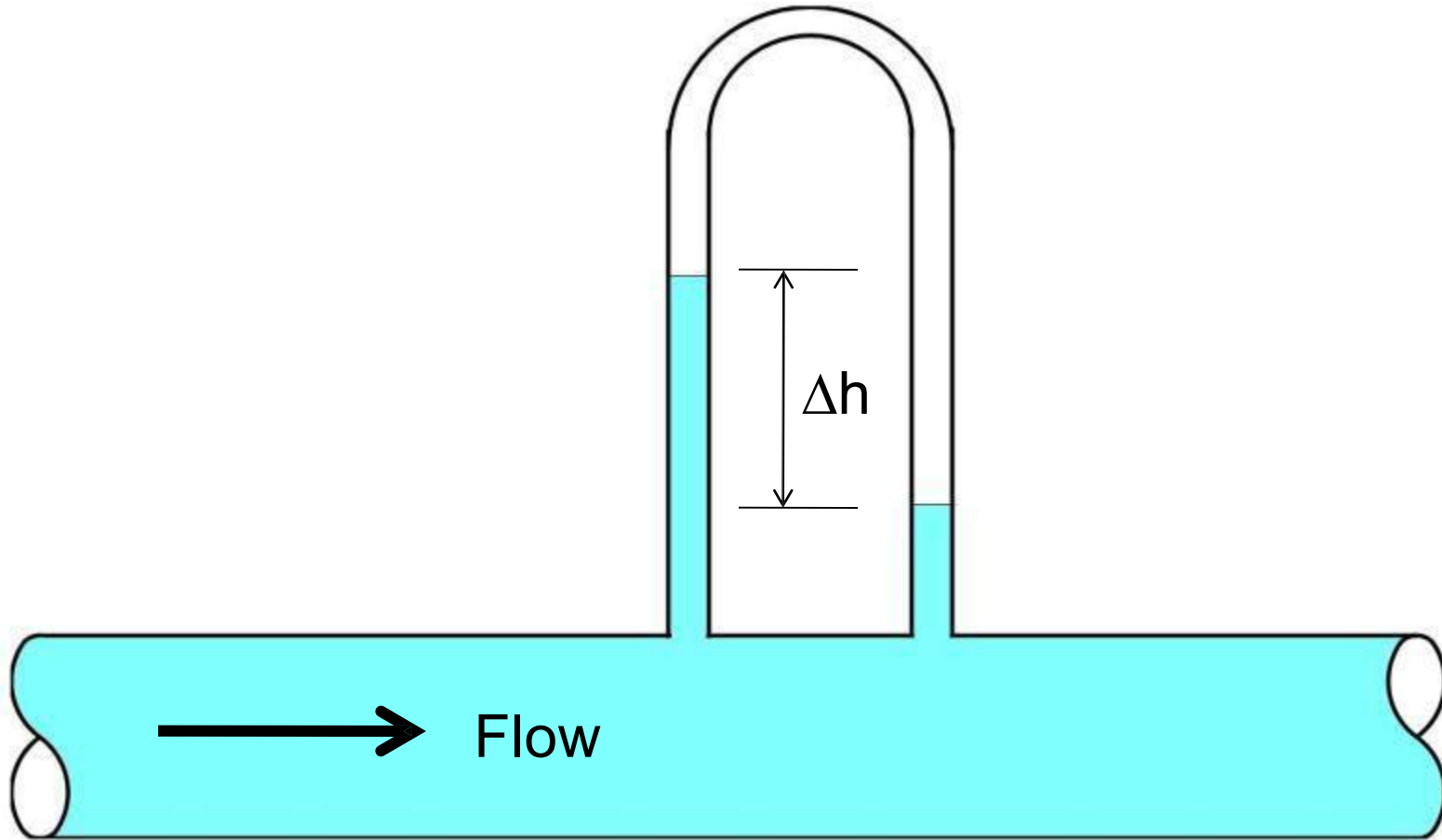
$$P_A - P_B = \rho_{\text{man}} g h$$



At the lower part of the pipe, require another manometer liquid to measure ΔP

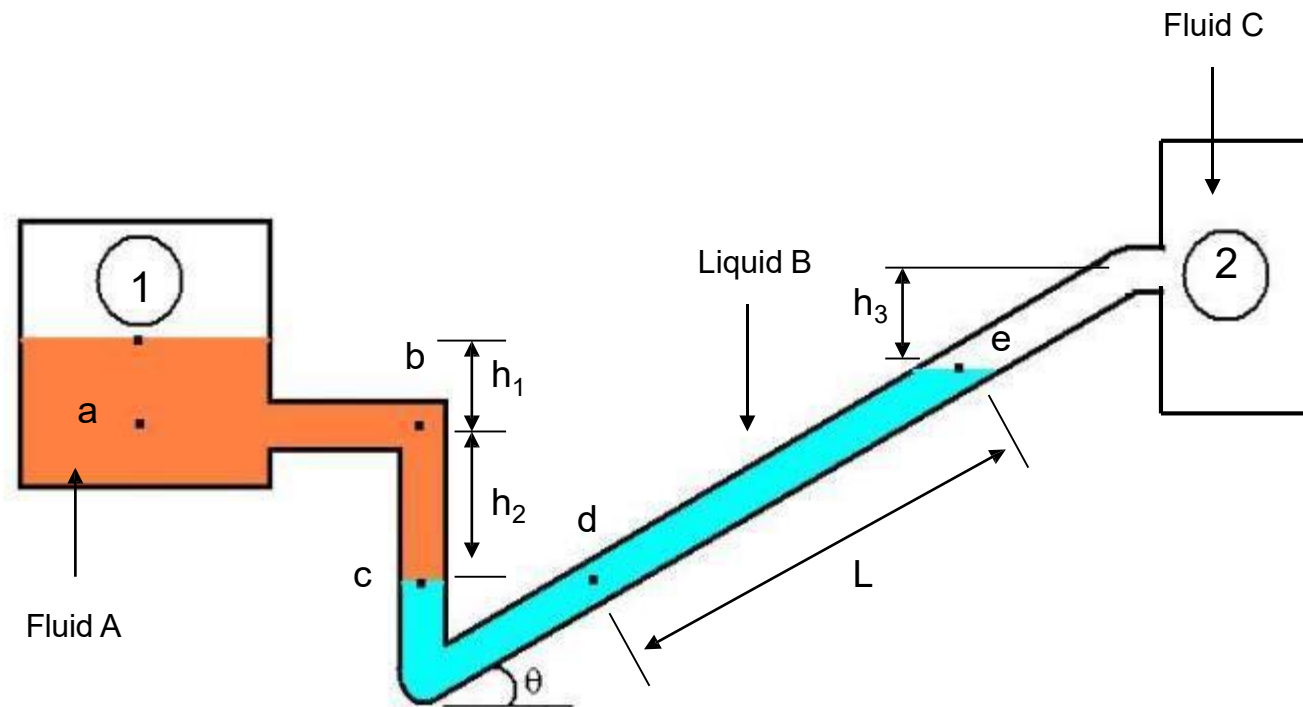


Prove that pressure difference
 $P_1 - P_2 = \Delta P = g \Delta h (\rho_{man} - \rho)$



At the upper part of the pipe, it does not require another manometer liquid to measure ΔP

Inclined-tube manometer



Prove that $P_1 - P_2 = \gamma_B L \sin \theta$

Selection of Manometer

Care must be taken when attaching the manometer to vessel, no burrs must be present around this joint. Burrs would alter the flow causing local pressure variations to affect the measurement.

Advantages and disadvantages of Manometer

Advantages

- small pressure differences can be measured
- They are very simple
- No calibration is required; the pressure difference can be calculated from first principles.

Disadvantages

- Not for measuring larger pressure differences
- Some liquids are unsuitable for use. Surface tension can also cause errors due to capillary rise; this can be avoided if the diameters of the tubes are sufficiently large - preferably not less than 15 mm diameter
- Slow response; unsuitable for measuring fluctuating pressures.

Class example 1

A simple U-tube manometer is installed across an orifice meter. The manometer is filled with mercury (specific gravity 13.6), and the liquid above the mercury is carbon tetrachloride (specific gravity 1.6). The manometer reads 200 mm. What is the pressure difference over the manometer?

Solution:

Pressure at X = pressure at X'

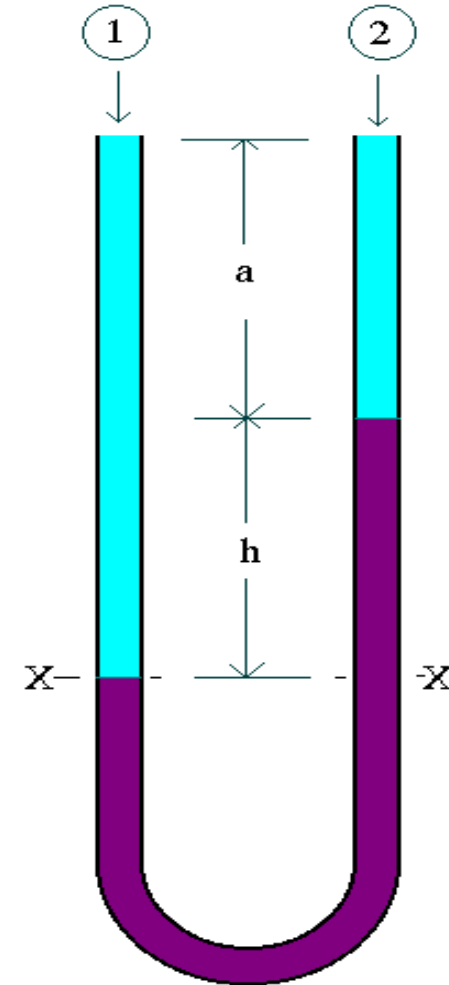
$$\text{Pressure at X, } P_X = P_1 + \rho_{\text{CCl}_4} g (a+h)$$

$$\text{Pressure at X', } P_{X'} = P_2 + \rho_{\text{mer}} g h + \rho_{\text{CCl}_4} g a$$

We can write.

$$\begin{aligned} P_1 + \rho_{\text{CCl}_4} g (a+h) &= P_2 + \rho_{\text{mer}} g h + \rho_{\text{CCl}_4} g a \\ \text{or, } P_1 - P_2 &= \rho_{\text{mer}} g h - \rho_{\text{CCl}_4} g h \\ &= g h \rho_{\text{water}} (\text{SG}_{\text{mercury}} - \text{SG}_{\text{CCl}_4}) \\ &= 9.81 \times 0.2 \times 1000 (13.6 - 1.6) \\ &= 23544 \text{ Pa} \end{aligned}$$

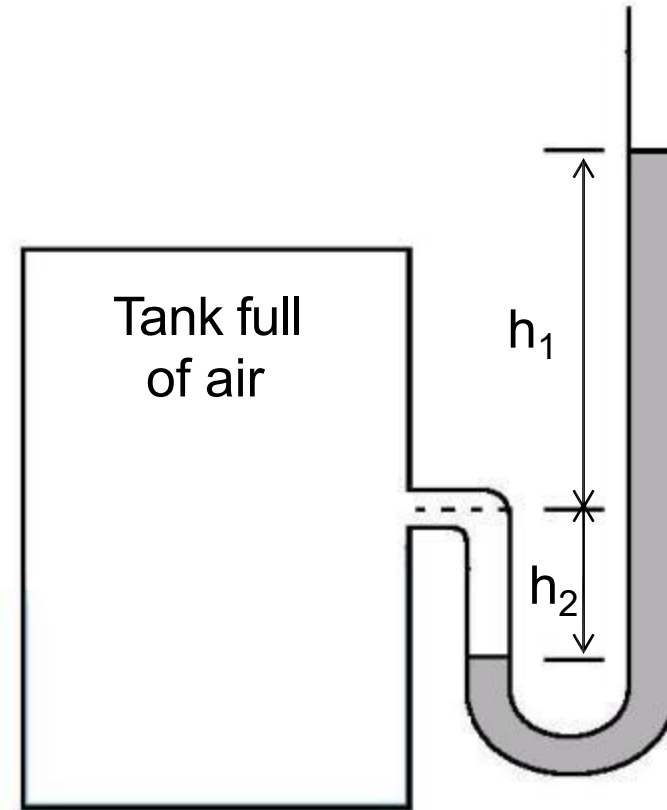
The pressure difference is 23544 Pa



Class example 2

The fluid shown shaded in the manometer is ethyl iodide with a specific gravity of 1.93. The heights are $h_1 = 100$ cm and $h_2 = 20$ cm.

- a) What is the gage pressure in the tank?
- b) What is the absolute pressure in the tank?



$$P_{\text{tank}} + \rho_{\text{air}}gh_2 - \rho_{\text{EI}}g(h_1 + h_2) = P_{\text{atm}}$$

Since we're taking gage pressure, $P_{\text{atm}} = 0$

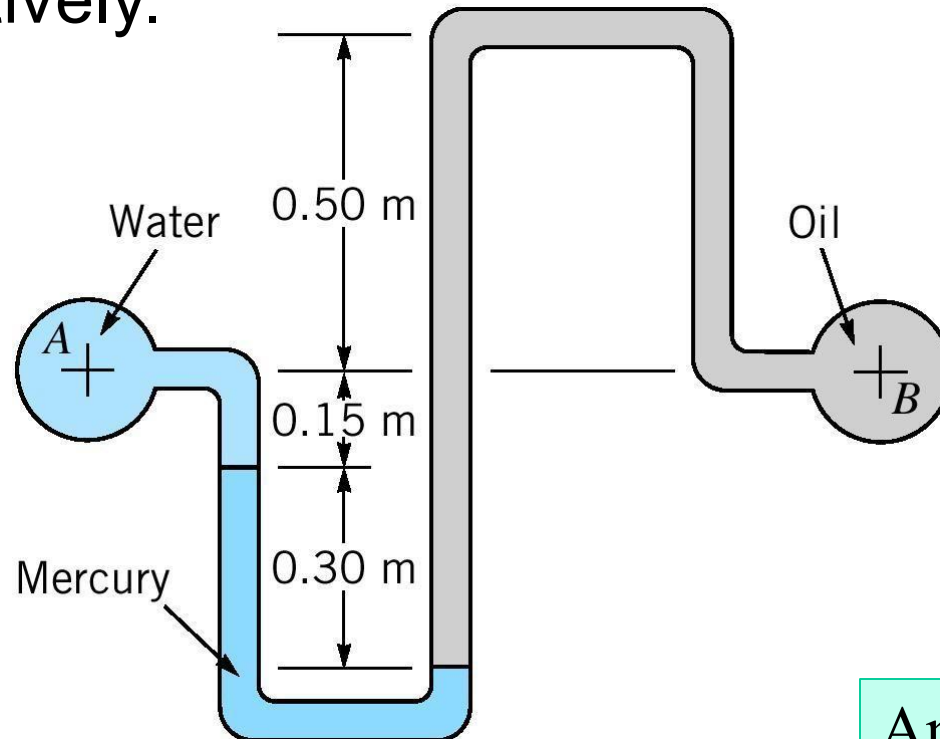
$$\begin{aligned} P_{\text{tank}} &= \rho_{\text{air}}gh_2 + \rho_{\text{EI}}g(h_1 + h_2) \\ &= -(1.2)(9.81)(0.2) + (1.93)(1000)(9.81)(1.00 + 0.200) \\ &= 22717.6 \text{ Pa} \\ &= 22.72 \text{ kPag} \end{aligned}$$

Assuming the density of air is too small

$$\begin{aligned} P_{\text{tank}} &= \rho_{\text{air}}gh_2 + \rho_{\text{EI}}g(h_1 + h_2) \\ &= (1.93)(1000)(9.81)(1.00 + 0.200) \\ &= 22719.96 \text{ Pa} \\ &= 22.72 \text{ kPag} \end{aligned}$$

Self assessment Assignment

- The mercury manometer below indicates a differential reading of 0.30 m when the pressure in pipe A is 30-mm Hg vacuum. Determine the pressure in pipe B if the specific gravity of the oil and mercury is 0.91 and 13.6, respectively.



Ans: 33.47 kPa

Mechanical and Electronic Pressure Measuring Device

- ❑ Measure of high pressures

- Pressure gage

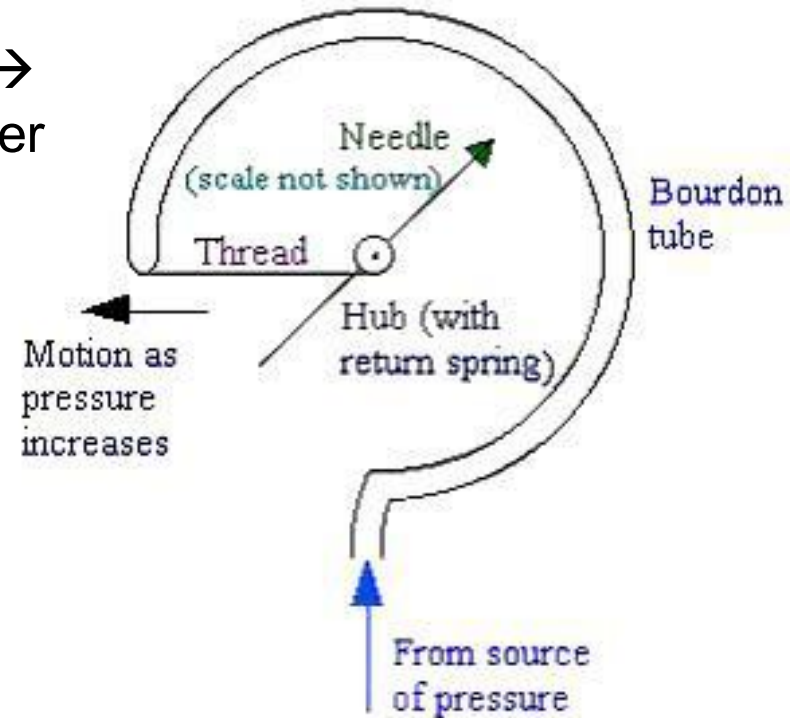
Used where only a visual indication is needed at the site where the pressure is being measured.

- Pressure transducer

Pressure is at a point, and the measured value is another point. displayed at

Bourdon Pressure Gage

- When pressure acts on an elastic structure, the structure will deform.
- This deformation relates to the magnitude of pressure
- As the pressure within tube increases → tube straighten → translated into pointer motion on dial.
- The pressure indicated
→ the difference between that communicated by the system to the external (ambient) pressure
→ P_{gage}



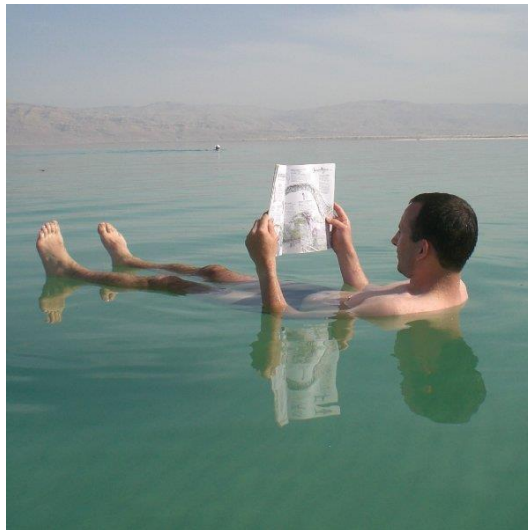
Pressure transducer

- The sensed pressure is converted to electrical signal
 - generated, and transmitted at another location such as central control station
 - Continuously monitor pressure changes with time (for rapidly changing pressure)
- Types of pressure transducers:
 - Strain gage pressure transducer
 - Linear-variable differential transformer (LVDT) pressure transducer
 - Piezoelectric pressure transducers
 - Quartz resonator pressure transducers

Buoyancy

Objectives:

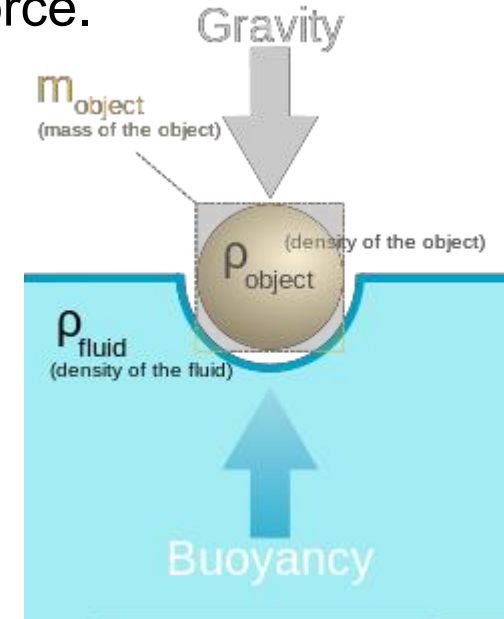
- To derive the equation of buoyant force
- To analyze the case of bodies floating on a fluid
- Use the principle of static equilibrium to solve forces involved in buoyancy problems.



What is Buoyancy

When an object is submerged or floating in a static fluid the **resultant force exerted** on it by the fluid is called buoyancy force.

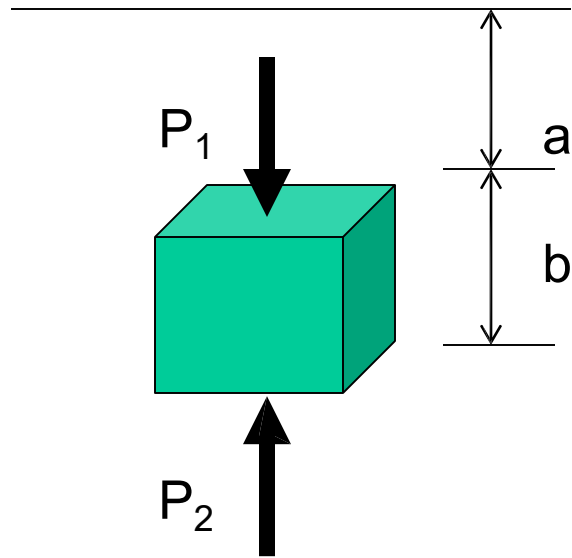
In a column of fluid, **pressure increases with depth** as a result of the weight of the overlying fluid. Thus a column of fluid, or an object submerged in the fluid, experiences greater pressure at the bottom of the column than at the top. This difference in pressure results in a net force that tends to accelerate an object upwards.



The **magnitude of buoyancy force is proportional** to the difference in the pressure between the top and the bottom of the column, and (as explained by Archimedes' principle) is also equivalent to the weight of the fluid that would otherwise occupy the column, *i.e.* the displaced fluid.

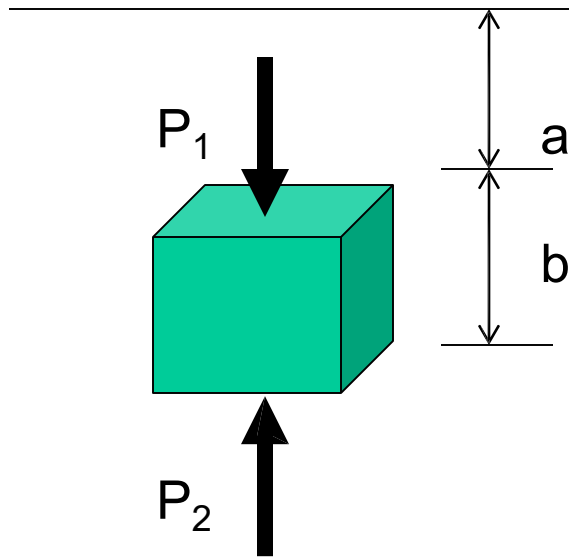
Archimedes' Principle: Any Object, wholly or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object

Buoyant force – Principle



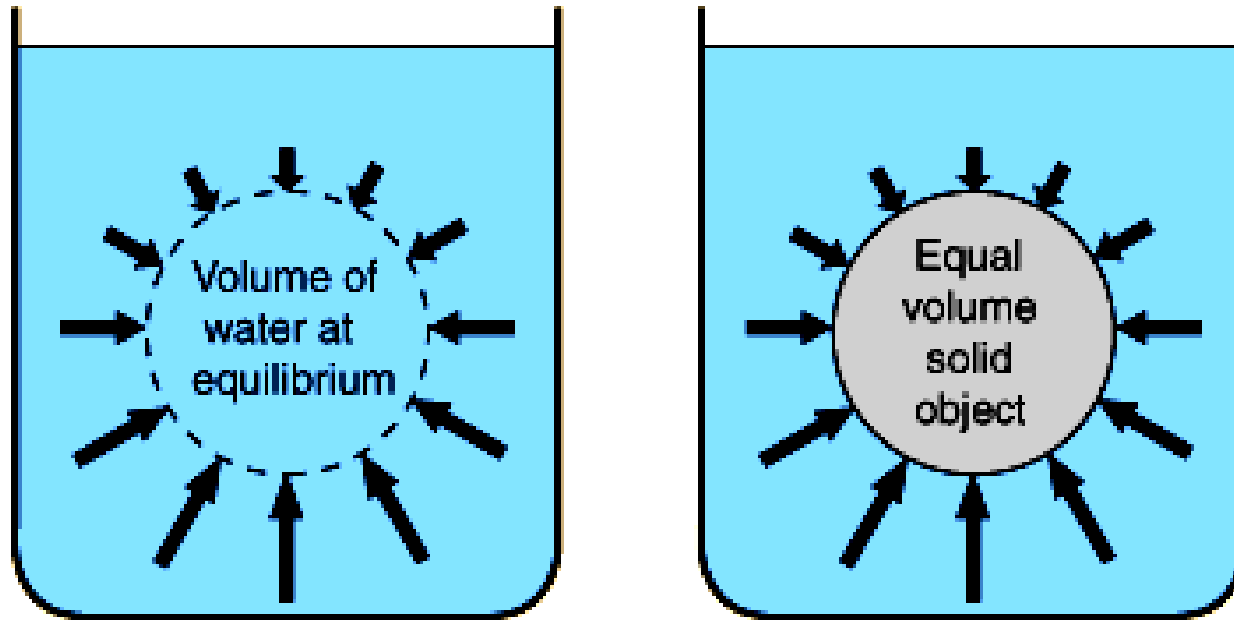
- The buoyant force is caused by the increase of pressure in a fluid with depth.
- Consider a body with a thickness, b is submerged in a liquid of density, ρ_f .
- The hydrostatic forces:
 $F_1 = P_1 A$ acting downward
 $F_2 = P_2 A$ acting upward

Buoyant force – Principle (Cont'd)



- Buoyant force = the difference between the two forces i.e the net upward force

$$\begin{aligned} F_B &= P_2 A - P_1 A \\ &= \rho_f g(a+b)A - \rho_f gaA \\ &= \rho_f gbA \\ &= \rho_f gV_b \end{aligned}$$



The **buoyant force** on the solid object is equal to the **weight of the fluid displaced** (Archimedes' principle)

For fully submerged or immersed body in a fluid ,
Archimedes' principle is restated as:

The buoyant force on a completely submerged body is equal to the weight of fluid displaced

$$F_B = W = \rho_f g V_b = \gamma_f V_b$$

Buoyant force = Weight of fluid displaced

With respect to the fluid dispersed:

$$F_B = W = \rho_b g V_f = \gamma_b V_f$$

$$F_B = \rho_f g V_{\text{body}}$$

$$= \rho_f g \left(\frac{m_{\text{body}}}{\rho_{\text{body}}} \right)$$

$$= m_{\text{body}} g \left(\frac{\rho}{\rho_{\text{body}}} \right)$$

Example

- A piece of irregularly shaped metal weighs 300.0 N in air. When the metal is completely submerged in water, it weighs 232.5 N. find the volume of the metal.

Solution:

$$F_B = W = 300 - 232.5 = 67.5 \text{ N}$$

$$F_B = \rho_f g V = (1000)(9.81) \quad V = 67.5$$

$$V = 0.00688 \text{ m}^3$$

For floating bodies, Archimedes' principle is restated as:

A floating body displaces a volume of fluid whose weight is exactly equal to its own

The weight of the entire body must be equal to the buoyant force

(Weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body).

- For floating bodies,

$$F_B = W$$

$$\Rightarrow \rho_f g V_{\text{body,sub}} = \rho_{\text{avg, body}} g V_{\text{total}}$$

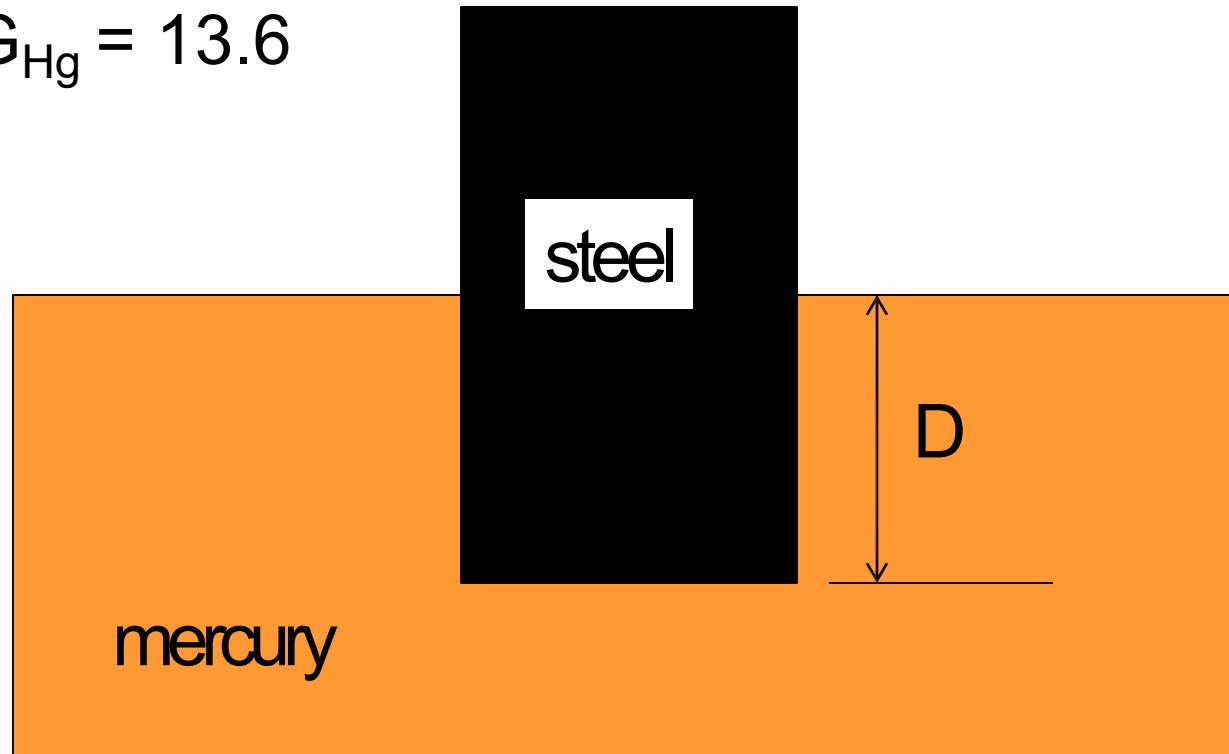
$$\frac{V_{\text{body,sub}}}{V_{\text{total}}} = \frac{\rho_{\text{avg, body}}}{\rho_f}$$

Example

- Determine the submerged depth of a cube of steel 0.30 m on each side floating in mercury.

Given $SG_{\text{steel}} = 7.8$

$SG_{\text{Hg}} = 13.6$



Solution

- $F_B = W$

$$V_{\text{body,total}} = 0.3 \times 0.3 \times 0.3 = 0.027 \text{ m}^3$$

$$\begin{aligned} \rho_f g V_{\text{body,sub}} &= \rho_{\text{body}} g V_{\text{body,total}} \\ (13.6)(1000)(9.81)(0.3 \times 0.3 \times D) &= \\ (7.8)(1000)(9.81)(0.027) \\ D &= 0.172 \text{ m} \end{aligned}$$

Example

- A solid block ($SG = 0.9$) floats such that 75% of its volume is in water and 25% of its volume is in liquid X, which is layered above the water. Determine the density of liquid X.

Solution

- $F_B = W$

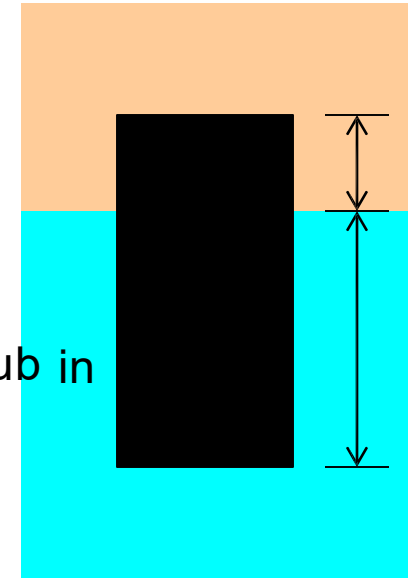
$$\rho_b g V_f = \rho_f g V_b$$

$$\rho_b g (V_w + V_X) = \rho_w g V_{b, \text{sub in } w} + \rho_X g V_{b, \text{sub in } X}$$

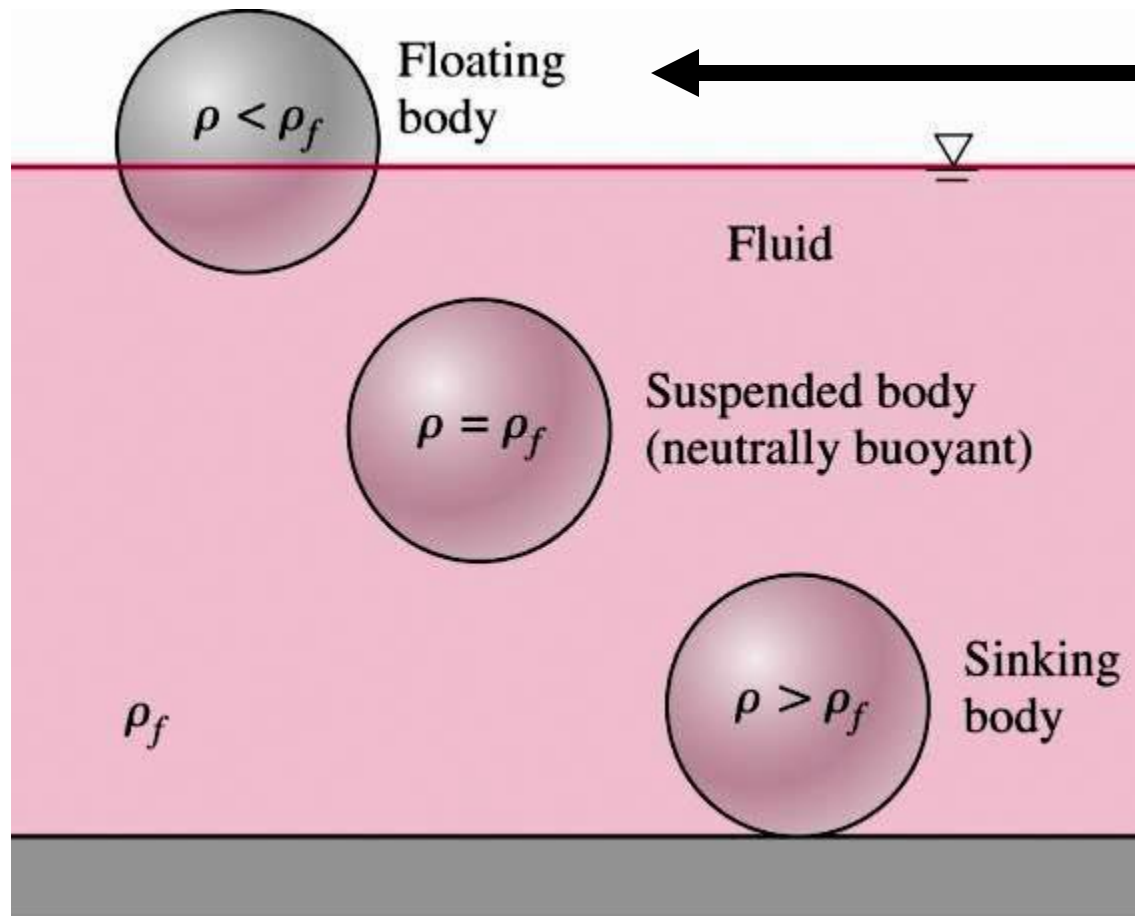
$$\rho_b V_b = \rho_w V_{b, \text{sub in } w} + \rho_X V_{b, \text{sub in } X}$$

$$0.9V_b = (1.0)(0.75V_b) + (SG_X)(0.25V_b)$$

$$SG_X = 0.60 \rightarrow \rho_X = 600 \text{ kg/m}^3$$

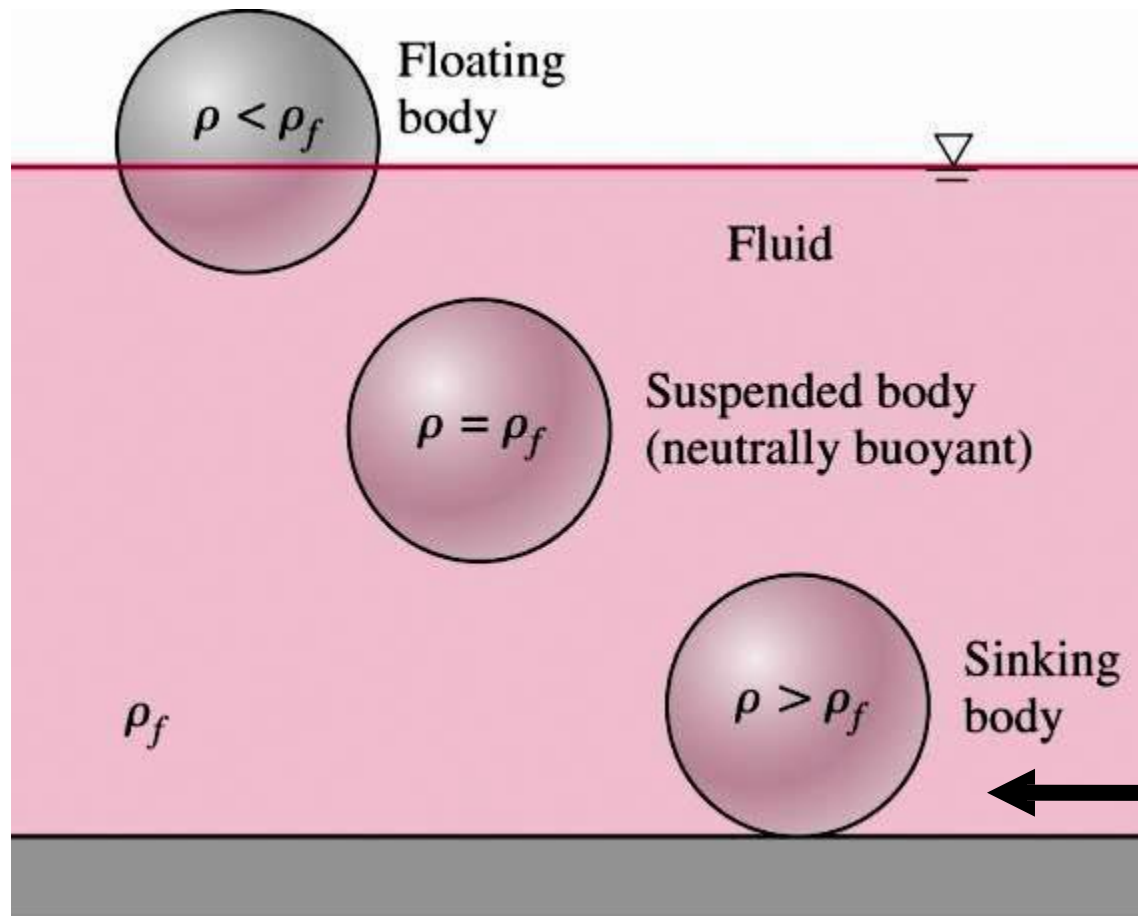


Sink or Float



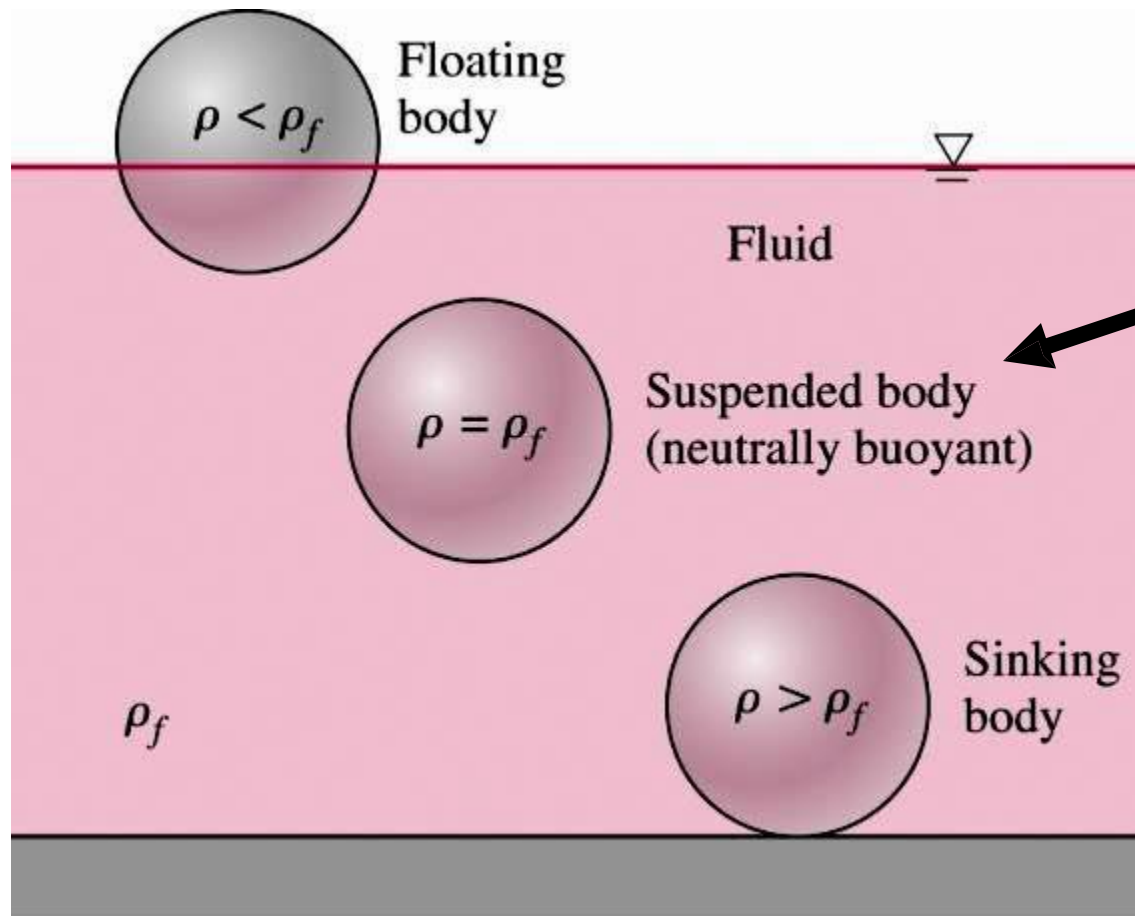
An object with an average specific weight less than that of the fluid tend to float because $W < F_B$

Sink or Float



An object with an average specific weight greater than that of the fluid tend to sink because $W > F_B$.

Sink or Float



An object whose average specific weight is equal to that of the fluid is neutrally buoyant.

What we learnt in this chapter (Outcomes)

- ✓ What is fluid statics and hydrostatic equilibrium?
- ✓ Pascal Law
- ✓ How to derive Barometric equation
- ✓ Definition of incompressible and compressible fluids?
- ✓ Pressure depth relationships for fluids
- ✓ Application of fluids statics
 - (a) pressure measurement using manometer
 - (b) Buoyancy force determination