

Soal dan jawaban

Tentukan turunan persial pertama fungsi-fungsi di bawah ini:

1. $z = 36 - x^2 - y^2$

Jawab

$$\frac{\partial z}{\partial x} = 0 - 2x - 0 = -2x$$

$$\frac{\partial z}{\partial y} = 0 - 0 - 2y = -2y$$

2. $z = xy - yz + xz$

Jawab

$$\frac{\partial z}{\partial x} = 1 \cdot y - 0 + 1 \cdot z = y + z$$

$$\frac{\partial z}{\partial y} = x \cdot 1 - 1 \cdot z + 0 = x - z$$

3. $F(x, y, z) = \sin(xy) - 2e^{xyz}$

Jawab

$$\frac{\partial F}{\partial x} = y \cos(xy) - 2yze^{xyz}$$

$$\frac{\partial F}{\partial x} = x \cos(xy) - 2xze^{xyz}$$

$$\frac{\partial F}{\partial z} = 0 - 2xye^{xyz} = -2xye^{xyz}$$

4. $z = xy^2 - 2x^2 + 3y^3$

$$\frac{\partial z}{\partial x} = 1 \cdot y^2 - 2 \cdot 2x + 0 = y^2 - 4x$$

$$\frac{\partial z}{\partial y} = x \cdot 2y - 0 + 3 \cdot 3y^2 = 2xy + 9y^2$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial x} (y^2 - 4x) = 0 - 4 = -4$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial y} \right) = \frac{\partial}{\partial y} (2xy + 9y^2) = 2x + 18y$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial y} (y^2 - 4x) = 2y$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = \frac{\partial}{\partial x} (2xy + 9y^2) = 2y$$

5. Daya yang dibutuhkan oleh resistor listrik dinyatakan dengan $P = \frac{E^2}{R}$ watt.

Jika $E = 200$ volt dan $R = 8$ Ohm. Dengan berapa besar daya berubah jika E menyusut 5 volt dan R menyusut dengan 2 Ohm.

$$P = \frac{E^2}{R}$$

$E = 200$ volt dan $R = 8$ Ohm

$\Delta E = -5$ volt sehingga $E + \Delta E = 195$

$\Delta R = -2$ ohm sehingga $R + \Delta R = 6$

Besar daya berubah

$$P = \frac{E^2}{R} = E^2 R^{-1} = \frac{200^2}{8} = 5000$$

$$P = E^2 R^{-1}$$

$$\begin{aligned} dP &= \frac{\partial P}{\partial E} dE + \frac{\partial P}{\partial R} dR = 2ER^{-1}dE - E^2R^{-2}dR \\ &= \frac{2E}{R}dE - \frac{E^2}{R^2}dR \\ &= \frac{2 \cdot 200}{8}(-5) - \frac{200^2}{8^2}(-2) = -250 + 1250 = 1000 \end{aligned}$$

Besar daya berubah menjadi

$$P = 5000 + 1000 = 6000$$

6. $f(x, y) = 1$ dengan $S = \{(x, y) : -1 \leq x \leq 2, x \leq y \leq x + 2\}$

integral lipat dan hasilnya

$$\begin{aligned} \int_{-1}^2 \int_x^{x+2} 1 \, dy \, dx &= \int_{-1}^2 y \Big|_x^{x+2} dx \\ &= \int_{-1}^2 (x+2) - x \, dx \\ &= \int_{-1}^2 2 \, dx \\ &= 2x \Big|_{-1}^2 \\ &= 2 \cdot 2 - 2 \cdot (-1) = 4 + 1 = 5 \end{aligned}$$

7. $f(x, y) = 1$ dengan $S = \{(x, y) : 0 \leq x \leq 2y, 1 \leq y \leq 4\}$

integral lipat dan hasilnya

$$\begin{aligned} \int_1^4 \int_0^{2y} 1 \, dx \, dy &= \int_1^4 x \Big|_0^{2y} dy \\ &= \int_1^4 2y - 0 \, dy \\ &= \int_1^4 2y \, dy \\ &= y^2 \Big|_1^4 \\ &= 4^2 - 1^2 = 16 - 1 = 15 \end{aligned}$$

8. Diberikan $f(x, y) = x + 2y$ dengan $S = \{(x, y) : 0 \leq x \leq y, 0 \leq y \leq 4\}$

tentukan integral lipat dan hasilnya

$$\begin{aligned} \int_0^4 \int_0^y x + 2y \, dx \, dy &= \int_0^4 \left(\frac{1}{2}x^2 + 2yx \right) \Big|_0^y dy \\ &= \int_0^4 \left(\frac{1}{2}y^2 + 2y \cdot y \right) - 0 \, dy \\ &= \int_0^4 \left(\frac{1}{2}y^2 + 2y^2 \right) dy \end{aligned}$$

$$\begin{aligned}
&= \int_0^4 \frac{5}{2} y^2 dy \\
&= \frac{5}{2} \cdot \frac{1}{3} y^3 \Big|_0^4 \\
&= \frac{5}{6} y^3 \Big|_0^4 \\
&= \frac{5}{6} 4^3 - 0 \\
&= \frac{5}{6} 4^3 = \frac{5}{6} 64 = \frac{5}{6} 64 = \frac{160}{3}
\end{aligned}$$

9. Fungsi $f(x, y) = 2x^2 - 5xy + y^3$, tentukan $\frac{\partial^2 f}{\partial x^2} - 2 \frac{\partial^2 f}{\partial y \partial x} + \frac{\partial^2 f}{\partial y^2}$

$$\frac{\partial z}{\partial x} = 4x - 5y + 0 = 4x - 5y$$

$$\frac{\partial z}{\partial y} = 0 - 5x + 3y^2 = -5x + 3y^2$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial x} (4x - 5y) = 4$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial y} \right) = \frac{\partial}{\partial y} (-5x + 3y^2) = 6y$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial y} (4x - 5y) = -5$$

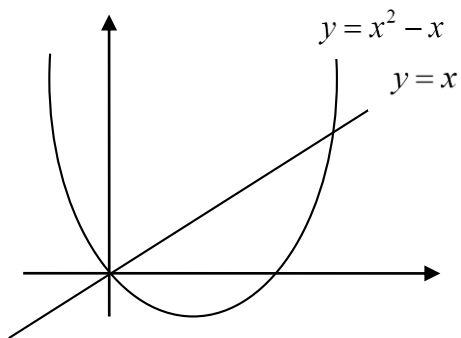
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = \frac{\partial}{\partial x} (-5x + 3y^2) = -5$$

$$\frac{\partial^2 f}{\partial x^2} - 2 \frac{\partial^2 f}{\partial y \partial x} + \frac{\partial^2 f}{\partial y^2} = 4 - 5 \cdot -5 + 6y = 4 + 25 + 6y = 29 + 6y$$

10. Tentukan

$$\begin{aligned}
\int_1^3 \int_0^{y^2} 2x + y^2 \, dx dy &= \int_1^3 x^2 + xy^2 \Big|_0^{y^2} dy \\
&= \int_1^3 ((y^2)^2 + y^2 \cdot y^2) - 0 \, dy \\
&= \int_1^3 2y^4 \, dy \\
&= \frac{2}{5} y^5 \Big|_1^3 = \frac{2}{5} \cdot 3^5 - \frac{2}{5} \cdot 1^5 = \frac{2}{5} \cdot 243 - \frac{2}{5} \cdot 1 = \frac{486}{5} - \frac{2}{5} = \frac{484}{5}
\end{aligned}$$

11. Tentukan luas daerah yang dibatasi oleh



Titik potong antara kurva $y = x^2 - x$ dan $y = x$

Adalah

$$y_1 = x^2 - x \quad \text{dan} \quad y_2 = x$$

$$y_1 = y_2$$

$$x^2 - x = x$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \text{ atau } x - 2 = 0$$

$$x = 0 \text{ atau } x = 2$$

Maka S himpunan y sederhana, $S = \{(x, y): 0 \leq x \leq 2, x^2 - x \leq y \leq x\}$

$$\begin{aligned} \text{Luas} &= \int_0^2 \int_{x^2-x}^x 1 \, dy \, dx = \int_0^2 y \Big|_{x^2-x}^x \, dx \\ &= \int_0^2 x - (x^2 - x) \, dx \\ &= \int_0^2 2x - x^2 \, dx \\ &= x^2 - \frac{1}{3}x^3 \Big|_0^2 \\ &= \left(2^2 - \frac{1}{3}2^3\right) - \left((0)^2 - \frac{1}{3}(0)^3\right) \\ &= \left(4 - \frac{8}{3}\right) - 0 = \frac{12}{3} - \frac{8}{3} = \frac{4}{3} \end{aligned}$$

