

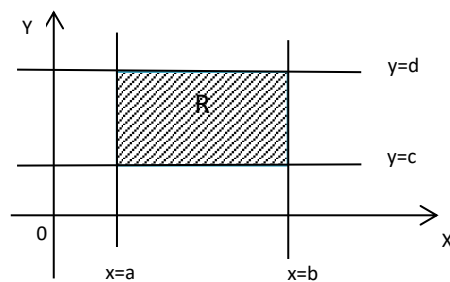
Pertemuan 6 KALKULUS III

LUAS

Luas daerah menggunakan integral lipat 2 untuk :

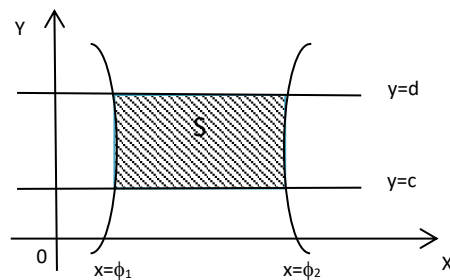
- persegi panjang $R = \{(x, y): a \leq x \leq b, c \leq y \leq d\}$

$$Luas = \iint_R 1 \, dA = \int_a^b \int_c^d 1 \, dydx = \int_c^d \int_a^b 1 \, dx dy$$



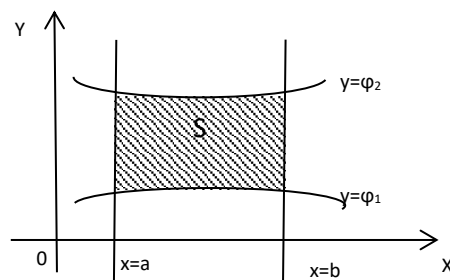
- S himpunan x sederhana, $S = \{(x, y): \phi_1 \leq x \leq \phi_2, c \leq y \leq d\}$

$$luas = \int_c^d \int_{\phi_1}^{\phi_2} 1 \, dx dy$$



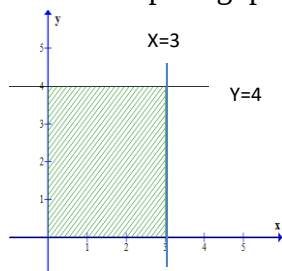
- S himpunan y sederhana, $S = \{(x, y): a \leq x \leq b, \varphi_1 \leq y \leq \varphi_2\}$

$$Luas = \int_a^b \int_{\varphi_1}^{\varphi_2} 1 \, dy dx$$



Contoh :

1. Tentukan luas persegi panjang dibawah ini :



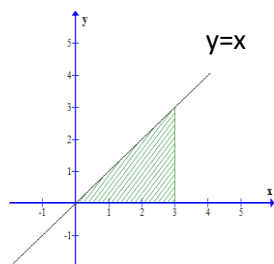
$$R = \{(x, y): 0 \leq x \leq 3, 0 \leq y \leq 4\}$$

$$Luas = \iint_R 1 \, dA = \int_0^3 \int_0^4 1 \, dy \, dx = \int_0^3 y \Big|_0^4 \, dx = \int_0^3 4 \, dx = 4x \Big|_0^3 = 4 \cdot 3 = 12$$

atau

$$Luas = \iint_R 1 \, dA = \int_0^4 \int_0^3 1 \, dx \, dy = \int_0^4 x \Big|_0^3 \, dy = \int_0^4 3 \, dy = 3y \Big|_0^4 = 3 \cdot 4 = 12$$

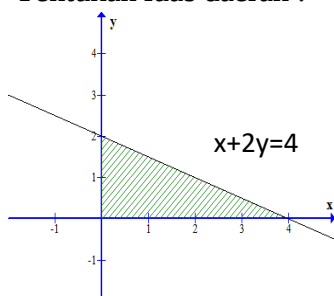
2. Tentukan luas daerah :



$$S \text{ himpunan } y \text{ sederhana, } S = \{(x, y): 0 \leq x \leq 3, 0 \leq y \leq x\}$$

$$Luas = \int_0^3 \int_0^x 1 \, dy \, dx = \int_0^3 y \Big|_0^x \, dx = \int_0^3 x \, dx = \frac{1}{2}x^2 \Big|_0^3 = \frac{1}{2} \cdot 3^2 = 4,5$$

3. Tentukan luas daerah :



$$\begin{aligned} x + 2y &= 4 \\ 2y &= 4 - x \\ y &= \frac{4-x}{2} \\ y &= 2 - \frac{1}{2}x \end{aligned}$$

Garis $x + 2y = 4$ memotong sumbu x ($y = 0$)

$$x + 2 \cdot 0 = 4$$

di $x = 4$, maka

$$S \text{ himpunan } y \text{ sederhana, } S = \{(x, y): 0 \leq x \leq 4, 0 \leq y \leq 2 - \frac{1}{2}x\}$$

$$Luas = \int_0^4 \int_0^{2-\frac{1}{2}x} 1 \, dy \, dx = \int_0^4 y \Big|_0^{2-\frac{1}{2}x} dx = \int_0^4 2 - \frac{1}{2}x \, dx = 2x - \frac{1}{4}x^2 \Big|_0^4 = 4$$

Atau

$$\text{Persamaan } x + 2y = 4$$

$$x = 4 - 2y$$

Garis $x + 2y = 4$ memotong sumbu y ($x = 0$)

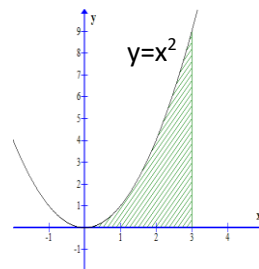
$$0 + 2y = 4$$

di $y = 2$, maka

S himpunan x sederhana, $S = \{(x, y): 0 \leq x \leq 4 - 2y, 0 \leq y \leq 2\}$

$$\begin{aligned} Luas &= \int_0^2 \int_0^{4-2y} 1 \, dx \, dy = \int_0^2 x \Big|_0^{4-2y} dy = \int_0^2 4 - 2y \, dy \\ &= 4y - y^2 \Big|_0^2 = 4 \end{aligned}$$

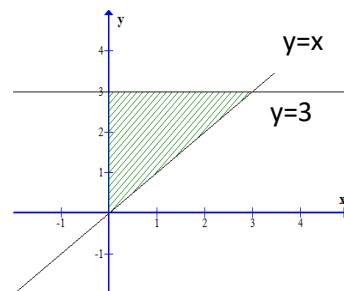
4. Tentukan luas daerah



S himpunan y sederhana, $S = \{(x, y): 0 \leq x \leq 3, 0 \leq y \leq x^2\}$

$$Luas = \int_0^3 \int_0^{x^2} 1 \, dy \, dx = \int_0^3 y \Big|_0^{x^2} dx = \int_0^3 x^2 \, dx = \frac{1}{3}x^3 \Big|_0^3 = \frac{1}{3} \cdot 3^3 = 9$$

5. Tentukan luas daerah



Titik potong antara garis $y = x$ dan $y = 3$ adalah $(3, 3)$, maka

S himpunan y sederhana, $S = \{(x, y): 0 \leq x \leq 3, x \leq y \leq 3\}$

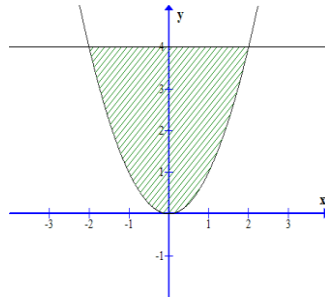
$$Luas = \int_0^3 \int_x^3 1 \, dy \, dx = \int_0^3 y \Big|_x^3 dx = \int_0^3 3 - x \, dx = 3x - \frac{1}{2}x^2 \Big|_0^3 = \frac{9}{2} = 4,5$$

Atau

S himpunan x sederhana, $S = \{(x, y): 0 \leq x \leq y, 0 \leq y \leq 3\}$

$$Luas = \int_0^3 \int_0^y 1 \, dx \, dy = \int_0^3 x \Big|_0^y dy = \int_0^3 y \, dy = \frac{1}{2}y^2 \Big|_0^3 = \frac{1}{2} \cdot 3^2 = 4,5$$

6. Tentukan luas daerah



Titik potong antara kurva $y = x^2$ dan $y = 4$ adalah

$$y = x^2 = 4$$

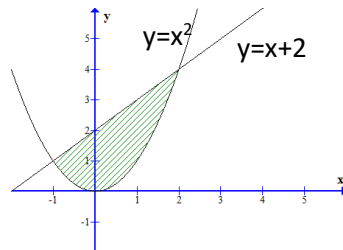
$$x^2 = 4$$

$$x = 2 \text{ atau } x = -2$$

Maka S himpunan y sederhana, $S = \{(x, y): -2 \leq x \leq 2, x^2 \leq y \leq 4\}$

$$\begin{aligned} \text{Luas} &= \int_{-2}^2 \int_{x^2}^4 1 \, dy dx = 2 \int_0^2 \int_{x^2}^4 1 \, dy dx = 2 \int_0^2 y \Big|_{x^2}^4 dx = 2 \int_0^2 (4 - x^2) dx \\ &= 2 \left(4x - \frac{1}{3}x^3 \right) \Big|_0^2 = 2 \left(4 \cdot 2 - \frac{1}{3} \cdot 2^3 \right) = 2 \left(8 - \frac{8}{3} \right) = \frac{32}{3} \end{aligned}$$

7. Tentukan luas daerah :



Titik potong antara kurva $y = x^2$ dan $y = x + 2$

Adalah

$$y_1 = x^2 \text{ dan } y_2 = x + 2$$

$$y_1 = y_2$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x - 2 = 0 \text{ atau } x + 1 = 0$$

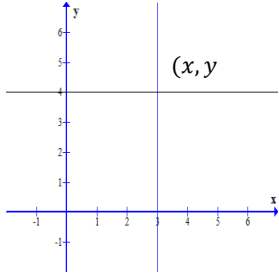
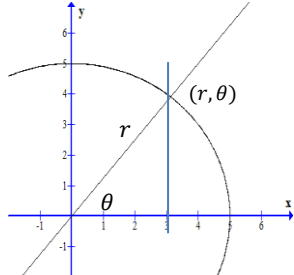
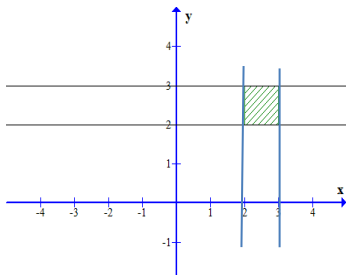
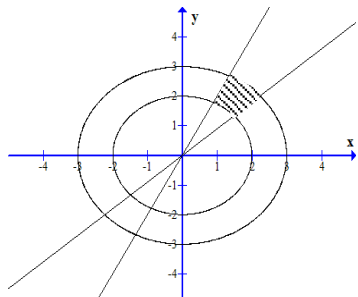
$$x = 2 \text{ atau } x = -1$$

Maka S himpunan y sederhana, $S = \{(x, y): -1 \leq x \leq 2, x^2 \leq y \leq x + 2\}$

$$\begin{aligned} \text{Luas} &= \int_{-1}^2 \int_{x^2}^{x+2} 1 \, dy dx = \int_{-1}^2 y \Big|_{x^2}^{x+2} dx = \int_{-1}^2 (x + 2 - x^2) dx \\ &= 2 \left(4x - \frac{1}{3}x^3 \right) \Big|_{-1}^2 = 2 \left(4 \cdot 2 - \frac{1}{3} \cdot 2^3 \right) = 2 \left(8 - \frac{8}{3} \right) = \frac{32}{3} \end{aligned}$$

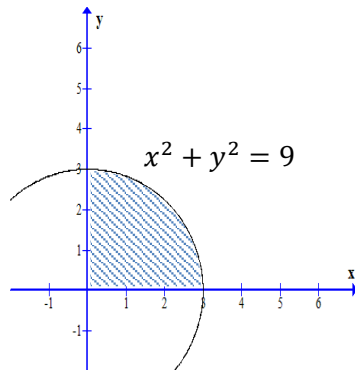
Luas dalam koordinat polar

Perubahan dari koordinat kartesius ke polar

Koordinat Kartesius	Koordinat polar
	
(x, y) $\{(x, y) -\infty < x < \infty, -\infty < y < \infty\}$	$(x, y) \equiv (r, \theta)$ $x = r \cos \theta$ dan $y = r \sin \theta$ $\{(r, \theta) 0 \leq r < \infty, 0 \leq \theta \leq 2\pi\}$ r bergerak dari titik $(0,0)$ keluar θ dari sumbu x positif ($0 = 0^\circ$) bergerak berlawanan dengan jarum jam sampai kembali ke sumbu x positif ($2\pi = 360^\circ$)
$dA = dx dy$ atau $dA = dy dx$	$dA = r dr d\theta$ atau $dA = r d\theta dr$
	
$\text{Luas} = \iint_R dA = \iint_R dx dy = \iint_R dy dx$	$\text{Luas} = \iint_S dA = \iint_S r dr d\theta = \iint_S r d\theta dr$

Contoh :

1. Tentukan luas daerah

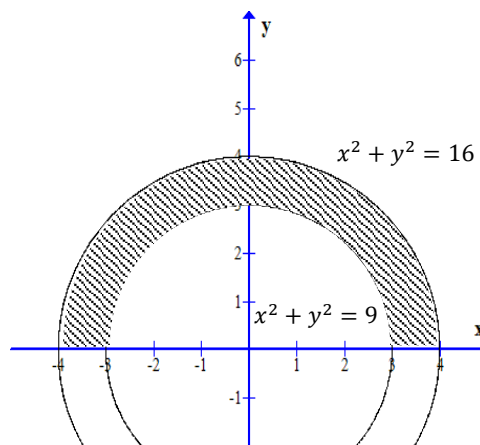


$$\{(r, \theta) | 0 \leq r \leq 3, 0 \leq \theta \leq \frac{\pi}{2}\}$$

$$luas = \int_0^3 \int_0^{\frac{\pi}{2}} r d\theta dr = \int_0^3 r \theta \Big|_0^{\frac{\pi}{2}} dr = \int_0^3 \frac{\pi}{2} r dr = \frac{\pi}{4} r^2 \Big|_0^3 = \frac{9}{4} \pi \quad \text{Atau}$$

$$luas = \int_0^{\frac{\pi}{2}} \int_0^3 r dr d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2 \Big|_0^3 d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} 3^2 d\theta = \int_0^{\frac{\pi}{2}} \frac{9}{2} d\theta = \frac{9}{2} \theta \Big|_0^{\frac{\pi}{2}} = \frac{9}{4} \pi$$

2. Tentukan luas daerah



$$\{(r, \theta) | 3 \leq r \leq 4, 0 \leq \theta \leq \pi\}$$

$$luas = \int_0^{\pi} \int_3^4 r dr d\theta = \int_0^{\pi} \frac{1}{2} r^2 \Big|_3^4 d\theta = \int_0^{\pi} \frac{1}{2} 4^2 - \frac{1}{2} 3^2 d\theta = \int_0^{\pi} \frac{7}{2} d\theta = \frac{7}{2} \theta \Big|_0^{\pi} = \frac{7}{2} \pi$$

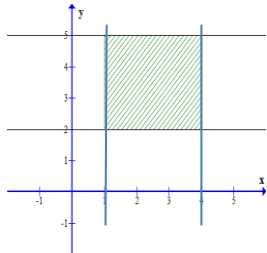
3. Tentukan luas $\{(r, \theta) : 0 \leq r \leq \cos \theta, 0 \leq \theta \leq \pi\}$

$$\begin{aligned} luas &= \int_0^{\pi} \int_0^{\cos \theta} r dr d\theta = \int_0^{\pi} \frac{1}{2} r^2 \Big|_0^{\cos \theta} d\theta = \int_0^{\pi} \frac{1}{2} \cos^2 \theta - 0 d\theta = \int_0^{\pi} \frac{1}{2} \cos^2 \theta d\theta \\ &= \int_0^{\pi} \frac{1}{2} \frac{1 + \cos 2\theta}{2} d\theta = \frac{1}{4} (\theta + \frac{1}{2} \sin 2\theta) \Big|_0^{\pi} \end{aligned}$$

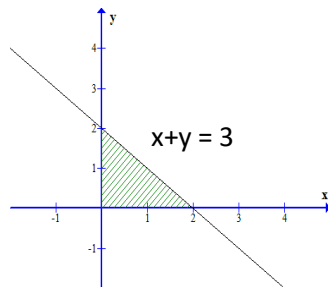
$$= \frac{1}{4}(\pi + \frac{1}{2} \sin 2\pi) - \frac{1}{4}(0 + \frac{1}{2} \sin 0) \Big|_0^{\pi} = \frac{1}{4}(\pi + 0) = \frac{\pi}{4}$$

Soal :

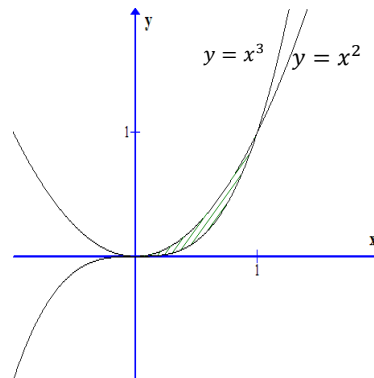
1. Tentukan luas daerah



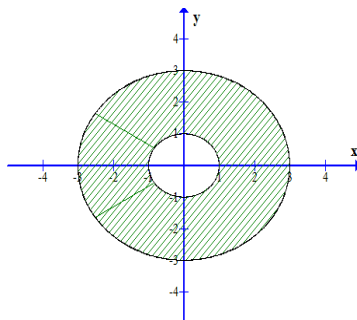
2. Tebtukan luas daerah



3. Tentukan luas daerah :



4. Tentukan luas daerah di arsir



5. Tentukan luas daerah yang dibatasi oleh kurva $y = x^2$ dan $y = x$

DAFTAR PUSTAKA : Purcell, E.J. & Vanberg, D.E. (1992). *Kalkulus dan Deometri Analitis*, Jilid 2. Jakarta: Erlangga.