ALJABAR LINIER

Sistem Persamaan Linier (Bagian 4)

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MATRIKS ADJOIN

Dalam matriks 2 x 2, matriks adjoinnya adalah pertukaran elemen-elemen yang terletak pada diagonal utama, dan mengalikan elemen-elemen yang terletak pada diagonal samping dengan –1.

$$\mathrm{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \Rightarrow \quad Adjoin\ (A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} 4 & -3 \\ 7 & 5 \end{bmatrix} \quad \Rightarrow \quad Adjoin\ (A) = \begin{bmatrix} 5 & 3 \\ -7 & 4 \end{bmatrix}$$

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MATRIKS ADJOIN

Matriks 3 x 3, matriks adjoinnya merupakan transpose dari kofaktor matriks A atau $(K_A)^T$. Dengan demikian:

Matriks A → Minor (A) → Kofaktor (A) → Transpose

$$\textbf{Adjoin (A)} = (\textbf{Kofaktor(A)})^{\mathbf{T}}$$

$$\text{Kofaktor (A)} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$\text{Adjoin (A)} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}^T = \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

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CONTOH 1

Hitunglah matriks adjoin dari matriks $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

Solusi

Dari perhitungan pertemuan sebelumnya, didapatkan matriks kofaktor A → $\text{Kofaktor (A)} = \begin{bmatrix} 0 & -3 & 2 \\ -4 & 2 & 0 \\ 4 & -1 & -2 \end{bmatrix}$

$$\text{Adjoin (A)} = \begin{bmatrix} 0 & -3 & 2 \\ -4 & 2 & 0 \\ 4 & -1 & -2 \end{bmatrix}^T = \begin{bmatrix} 0 & -4 & 4 \\ -3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

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INVERS MATRIKS

Invers matriks $\boldsymbol{A}$ atau dituliskan $\boldsymbol{A^{-1}}$adalah sebuah kebalikan (invers) dari sebuah matriks $\boldsymbol{A}$ yang memenuhi:

$$\boldsymbol{A.A^{-1} = A^{-1}.A = I}$$

Dengan: I adalah matriks indentitas n x n dimana diagonal utama bernilai 1 dan elemen-elemen lainnya bernilai 0, contoh matriks indentitas 3 x 3:

$$\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{invers(A) = A^{-1} = \frac{1}{det(A)}\, Adjoin\ (A)}$$

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CONTOH 2

Hitunglah invers matriks dari matriks $\mathrm{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

Solusi

Dari perhitungan pertemuan sebelumnya, didapatkan det (A) $|\mathrm{A}| = -4$

Dari perhitungan sebelumnya, didapatkan matriks adjoin A $\mathrm{Adjoin\ (A)} = \begin{bmatrix} 0 & -4 & 4 \\ -3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$

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CONTOH 2

$$A^{-1} = \frac{1}{\det(A)} \text{ Adjoin } (A)$$

$$A^{-1} = \frac{1}{-4}\begin{bmatrix} 0 & -4 & 4 \\ -3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{-4}(-4) & \frac{1}{-4}\cdot 4 \\ \frac{1}{-4}(-3) & \frac{1}{-4}\cdot 2 & \frac{1}{-4}(-1) \\ \frac{1}{-4}\cdot 2 & \frac{1}{-4}\cdot 0 & \frac{1}{-4}(-2) \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Atau:

$$A^{-1} = -0{,}25\begin{bmatrix} 0 & -4 & 4 \\ -3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0{,}25(-4) & -0{,}25\cdot 4 \\ -0{,}25(-3) & -0{,}25\cdot 2 & -0{,}25(-1) \\ -0{,}25\cdot 2 & -0{,}25\cdot 0 & -0{,}25(-2) \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 0{,}75 & -0{,}5 & 0{,}25 \\ -0{,}5 & 0 & 0{,}5 \end{bmatrix}$$

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SISTEM PERSAMAAN LINIER

Suatu sistem persamaan linier yang terdiri dari n persamaan dan n variabel yang tidak diketahui, dapat digambarkan dalam bentuk umum:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = a_{1n+1}$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = a_{2n+1}$$

$$\ldots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_3 = a_{nn+1}$$

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{1n+1} \\ a_{1n+1} \\ \ldots \\ a_{nn+1} \end{bmatrix}$$

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SISTEM PERSAMAAN LINIER

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ \dots \\ b_{n1} \end{bmatrix}$$

A . X = B

Penyelesaian dari SPL adalah matrik X

$$\mathbf{A^{-1}.A.X = A^{-1}.B}$$

$$\mathbf{I.X = A^{-1}.B}$$

$$\mathbf{X = A^{-1}.B}$$

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CONTOH 3

$$x_1 + 2x_2 + 1x_3 = 4$$
$$2x_1 + 2x_2 + 3x_3 = 7$$
$$x_1 + 2x_2 + 3x_3 = 6$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 7 \\ 6 \end{bmatrix}$$

Dari perhitungan sebelumnya didapatkan A^{-1} adalah:

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\mathbf{X = A^{-1}.B}$$

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CONTOH 3

$$X = A^{-1}.B \;=\begin{bmatrix}0 & 1 & -1\\ \frac{3}{4} & -\frac{1}{2} & \frac{1}{4}\\ -\frac{1}{2} & 0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}4\\7\\6\end{bmatrix}$$

$$=\begin{bmatrix}0\cdot 4+1\cdot 7+(-1)\cdot 6\\ \frac{3}{4}\cdot 4+\left(-\frac{1}{2}\right)\cdot 7+\frac{1}{4}\cdot 6\\ -\frac{1}{2}\cdot 4+0\cdot 7+\frac{1}{2}\cdot 6\end{bmatrix}$$

$$=\begin{bmatrix}0+7-6\\ 3-\frac{7}{2}+\frac{3}{2}\\ -2+0+3\end{bmatrix}=\begin{bmatrix}1\\3-2\\1\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\mathbf{x_1} = \mathbf{1}$

$\mathbf{x_2} = \mathbf{1}$

$\mathbf{x_3} = \mathbf{1}$

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CONTOH 3 (pilihan cara desimal)

ATAU:

$$A^{-1}=\begin{bmatrix}0 & 1 & -1\\ \frac{3}{4} & -\frac{1}{2} & \frac{1}{4}\\ -\frac{1}{2} & 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}0 & 1 & -1\\ 0{,}75 & -0{,}5 & 0{,}25\\ -0{,}5 & 0 & 0{,}5\end{bmatrix}$$

$$X = A^{-1}.B$$

$$=\begin{bmatrix}0 & 1 & -1\\ 0{,}75 & -0{,}5 & 0{,}25\\ -0{,}5 & 0 & 0{,}5\end{bmatrix}\begin{bmatrix}4\\7\\6\end{bmatrix}$$

$$=\begin{bmatrix}0\cdot 4+1\cdot 7+(-1)\cdot 6\\ 0{,}75\cdot 4+(-0{,}5)\cdot 7+0{,}25\cdot 6\\ -0{,}5\cdot 4+0\cdot 7+0{,}5\cdot 6\end{bmatrix}=\begin{bmatrix}0+7-6\\ 3-3{,}5+1{,}5\\ -2+0+3\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

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CONTOH 4

$$
\begin{aligned}
-2x_1 + 5x_2 + 4x_3 &= 2 \\
3x_1 + x_2 + 2x_3 &= 4 \\
-x_1 + 2x_2 + 2x_3 &= 8
\end{aligned}
\quad \Rightarrow \quad
\begin{bmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =
\begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}
\quad \Rightarrow \quad
A = \begin{bmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}, \quad
B = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}
$$

Minor (A) → Kofaktor (A) → Det (A) / Adjoin (A) → Invers (A) → Nilai X

CONTOH 4

Minor (A)

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

$$m_{11} = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2 - 4 = -2$$

$$m_{12} = \begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} = 6 + 2 = 8$$

$$m_{13} = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} = 6 + 1 = 7$$

$$m_{21} = \begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix} = 10 - 8 = 2$$

$$m_{22} = \begin{vmatrix} -2 & 4 \\ -1 & 2 \end{vmatrix} = -4 + 4 = 0$$

$$m_{23} = \begin{vmatrix} -2 & 5 \\ -1 & 2 \end{vmatrix} = -4 + 5 = 1$$

$$m_{31} = \begin{vmatrix} 5 & 4 \\ 1 & 2 \end{vmatrix} = 10 - 4 = 6$$

$$m_{32} = \begin{vmatrix} -2 & 4 \\ 3 & 2 \end{vmatrix} = -4 - 12 = -16$$

$$m_{33} = \begin{vmatrix} -2 & 5 \\ 3 & 1 \end{vmatrix} = -2 - 15 = -17$$

$$\text{Minor}(A) = \begin{bmatrix} -2 & 8 & 7 \\ 2 & 0 & 1 \\ 6 & -16 & -17 \end{bmatrix}$$

CONTOH 4

$$\text{Minor (A)} = \begin{bmatrix} -2 & 8 & 7 \\ 2 & 0 & 1 \\ 6 & -16 & -17 \end{bmatrix}$$

$$\text{Kofaktor (A)} = \begin{bmatrix} -2 & -8 & 7 \\ -2 & 0 & -1 \\ 6 & 16 & -17 \end{bmatrix}$$

Det (A)

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

$$|A| = (-2 \cdot 3) + (0 \cdot 1) + (-1 \cdot 2)$$

$$= -6 - 2 \quad = -8$$

Adjoin (A)

$$\text{Adjoin (A)} = \begin{bmatrix} -2 & -8 & 7 \\ -2 & 0 & -1 \\ 6 & 16 & -17 \end{bmatrix}^{T} = \begin{bmatrix} -2 & -2 & 6 \\ -8 & 0 & 16 \\ 7 & -1 & -17 \end{bmatrix}$$

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CONTOH 4

Invers (A)

$$A^{-1} = \frac{1}{\det(A)} \text{ Adjoin (A)}$$

$$A^{-1} = \frac{1}{-8} \begin{bmatrix} -2 & -2 & 6 \\ -8 & 0 & 16 \\ 7 & -1 & -17 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{-8}(-2) & \frac{1}{-8}(-2) & \frac{1}{-8} \cdot 6 \\ \frac{1}{-8}(-8) & \frac{1}{-8} \cdot 0 & \frac{1}{-8} \cdot 16 \\ \frac{1}{-8} \cdot 7 & \frac{1}{-8}(-1) & \frac{1}{-8}(-17) \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \\ 1 & 0 & -2 \\ -\frac{7}{8} & \frac{1}{8} & \frac{17}{8} \end{bmatrix}$$

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CONTOH 4

$$X = A^{-1}.B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4,5 \\ -14 \\ 15,75 \end{bmatrix}$$

x_1 = –4,5

x_2 = – 14

x_3 = 15,75

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \\ 1 & 0 & -2 \\ -\frac{7}{8} & \frac{1}{8} & \frac{17}{8} \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 4 + (-\frac{3}{4}) \cdot 8 \\ 1 \cdot 2 + 0 \cdot 4 + (-2) \cdot 8 \\ -\frac{7}{8} \cdot 2 + \frac{1}{8} \cdot 4 + \frac{17}{8} \cdot 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{2}{2} - \frac{12}{2} \\ 2 - 16 \\ -\frac{7}{4} + \frac{2}{4} + \frac{68}{4} \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} \\ -14 \\ \frac{63}{4} \end{bmatrix} = \begin{bmatrix} -4,5 \\ -14 \\ 15,75 \end{bmatrix}$$

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CONTOH 4 (pilihan cara desimal)

Invers (A)

$$A^{-1} = \frac{1}{\det(A)} \text{ Adjoin } (A)$$

$$A^{-1} = \frac{1}{-8} \begin{bmatrix} -2 & -2 & 6 \\ -8 & 0 & 16 \\ 7 & -1 & -17 \end{bmatrix} = -0,125 \begin{bmatrix} -2 & -2 & 6 \\ -8 & 0 & 16 \\ 7 & -1 & -17 \end{bmatrix}$$

$$= \begin{bmatrix} -0,125(-2) & -0,125(-2) & -0,125 \cdot 6 \\ -0,125(-8) & -0,125 \cdot 0 & -0,125 \cdot 16 \\ -0,125 \cdot 7 & -0,125(-1) & -0,125(-17) \end{bmatrix} = \begin{bmatrix} 0,25 & 0,25 & -0,75 \\ 1 & 0 & -2 \\ -0,875 & 0,125 & 2,125 \end{bmatrix}$$

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CONTOH 4 (pilihan cara desimal)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4{,}5 \\ -14 \\ 15{,}75 \end{bmatrix}$$

$x_1 = -4{,}5$

$x_2 = -14$

$x_3 = 15{,}75$

$$X = A^{-1}.B$$

$$= \begin{bmatrix} 0{,}25 & 0{,}25 & -0{,}75 \\ 1 & 0 & -2 \\ -0{,}875 & 0{,}125 & 2{,}125 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}25 \cdot 2 + 0{,}25 \cdot 4 + (-0{,}75) \cdot 8 \\ 1 \cdot 2 + 0 \cdot 4 + (-2) \cdot 8 \\ -0{,}875 \cdot 2 + 0{,}125 \cdot 4 + 2{,}125 \cdot 8 \end{bmatrix} = \begin{bmatrix} 0{,}5 + 1 - 6 \\ 2 - 16 \\ -1{,}75 + 0{,}5 + 17 \end{bmatrix} = \begin{bmatrix} -4{,}5 \\ -14 \\ 15{,}75 \end{bmatrix}$$

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METODE CRAMER

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ \dots \\ b_{n1} \end{bmatrix}$$

A . X = B

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_{12} & \dots & a_{1n} \\ a_{21} & b_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & b_{n2} & \dots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}}$$

$$x_1 = \frac{\begin{vmatrix} b_{11} & a_{12} & \dots & a_{1n} \\ b_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}}$$

$$x_n = \frac{\begin{vmatrix} a_{11} & a_{12} & \dots & b_{1n} \\ a_{21} & a_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & b_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}}$$

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CONTOH 5

$$3x + 2y = 13000$$
$$x + 3y = 9000$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13000 \\ 9000 \end{bmatrix}$$

$$\mathbf{A} \quad \mathbf{X} = \mathbf{B}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

Nilai x

$$\mathbf{x} = \frac{\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

Nilai y

$$\mathbf{y} = \frac{\begin{vmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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CONTOH 5

$$x = \frac{\begin{vmatrix} \mathbf{13000} & 2 \\ \mathbf{9000} & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}}$$

$$= \frac{(13000 \times 3) - (9000 \times 2)}{(3 \times 3) - (1 \times 2)}$$

$$= \frac{39000 - 18000}{9 - 2}$$

$$= \frac{21000}{7}$$

$$= 3000$$

$$y = \frac{\begin{vmatrix} 3 & \mathbf{13000} \\ 1 & \mathbf{9000} \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}}$$

$$= \frac{(3 \times 9000) - (1 \times 13000)}{(3 \times 3) - (1 \times 2)}$$

$$= \frac{27000 - 13000}{9 - 2}$$

$$= \frac{14000}{7}$$

$$= 2000$$

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CONTOH 6

$$\begin{aligned} -2x_1 + 5x_2 + 4x_3 &= 2 \\ 3x_1 + x_2 + 2x_3 &= 4 \\ -x_1 + 2x_2 + 2x_3 &= 8 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} \mathbf{2} & 5 & 4 \\ \mathbf{4} & 1 & 2 \\ \mathbf{8} & 2 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} -2 & \mathbf{2} & 4 \\ 3 & \mathbf{4} & 2 \\ -1 & \mathbf{8} & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}} \qquad x_3 = \frac{\begin{vmatrix} -2 & 5 & \mathbf{2} \\ 3 & 1 & \mathbf{4} \\ -1 & 2 & \mathbf{8} \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}}$$

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CONTOH 6

$$- \quad + \quad -$$

$$x_1 = \frac{\begin{vmatrix} 2 & 5 & 4 \\ 4 & 1 & 2 \\ 8 & 2 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}} = \frac{-4\begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 8 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 8 & 2 \end{vmatrix}}{-1\begin{vmatrix} 5 & 4 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} -2 & 4 \\ 3 & 2 \end{vmatrix} + 2\begin{vmatrix} -2 & 5 \\ 3 & 1 \end{vmatrix}}$$

$$+ \quad - \quad +$$

$$= \frac{-4(10-8)+1(4-32)-2(4-40)}{-1(10-4)-2(-4-12)+2(-2-15)}$$

$$= \frac{-4(2)+1(-28)-2(-36)}{-1(6)-2(-16)+2(-17)}$$

$$= \frac{-8-28+72}{-6+32-34} = \frac{36}{-8} = -4,5$$

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CONTOH 6

$$x_2 = \frac{\begin{array}{ccc} + & - & + \\ \end{array} \begin{vmatrix} -2 & 2 & 4 \\ 3 & 4 & 2 \\ -1 & 8 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}} = \frac{-2\begin{vmatrix} 4 & 2 \\ 8 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} + 4\begin{vmatrix} 3 & 4 \\ -1 & 8 \end{vmatrix}}{-8}$$

$$= \frac{-2(8-16) - 2(6+2) + 4(24+4)}{-8}$$

$$= \frac{-2(-8) - 2(8) + 4(28)}{-8}$$

$$= \frac{16 - 16 + 112}{-8} = \frac{112}{-8} = -14$$

CONTOH 6

$$x_3 = \frac{\begin{array}{ccc} + & - & + \\ \end{array} \begin{vmatrix} -2 & 5 & 2 \\ 3 & 1 & 4 \\ -1 & 2 & 8 \end{vmatrix}}{\begin{vmatrix} -2 & 5 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix}} = \frac{-2\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} - 5\begin{vmatrix} 3 & 4 \\ -1 & 8 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}}{-8}$$

$$= \frac{-2(8-8) - 5(24+4) + 2(6+1)}{-8}$$

$$= \frac{-2(0) - 5(28) + 2(7)}{-8}$$

$$= \frac{-140 + 14}{-8} = \frac{-126}{-8} = 15{,}75$$