PENGANTAR TEKNIK ELEKTRO

NODAL ANALYSIS
Bagian 1

Wike Handini

KONDUKTANSI G

$$G = \frac{1}{R}$$

Dengan:

G = konduktansi (mho atau ℧)

R = resistansi (ohm atau Ω)

Rangkaian paralel konduktansi

$$G_{eq} = G_1 + G_2 + \cdots + G_n = \sum_{n=1}^{N} G_n$$

Rangkaian seri konduktansi

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_N}$$

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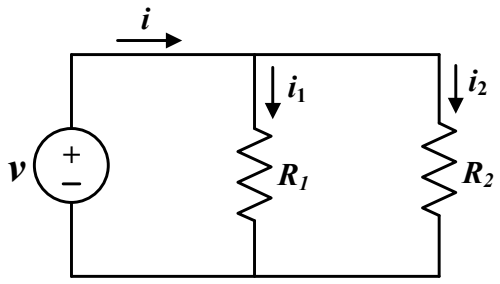
KONDUKTANSI G → PRINSIP PEMBAGI ARUS



Dengan: $G_1 = \frac{1}{R_1}$ dan $G_2 = \frac{1}{R_2}$

maka

$$i_1 = \frac{R_2 i}{R_1 + R_2} \Rightarrow i_1 = \frac{G_1 i}{G_1 + G_2}$$

$$i_2 = \frac{R_1 i}{R_1 + R_2} \Rightarrow i_2 = \frac{G_2 i}{G_1 + G_2}$$

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

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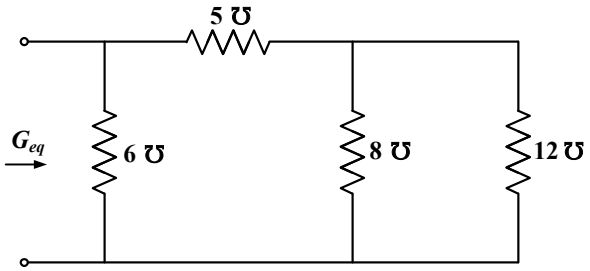
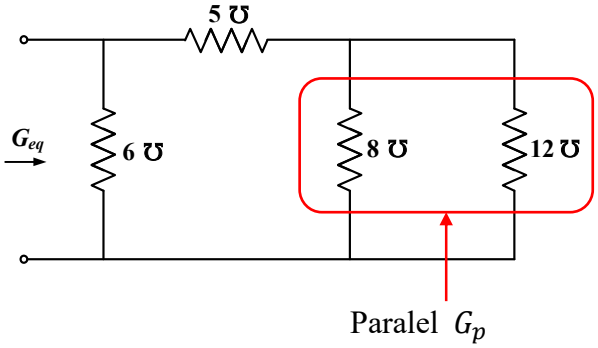
CONTOH 1

Hitunglah G_{eq} dari rangkaian



Solusi



Paralel G_p

$G_p = 8 + 12$

$= 20$ Ʊ

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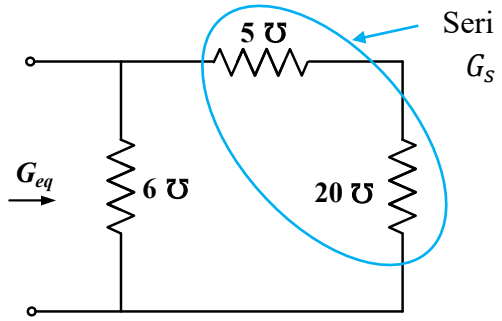
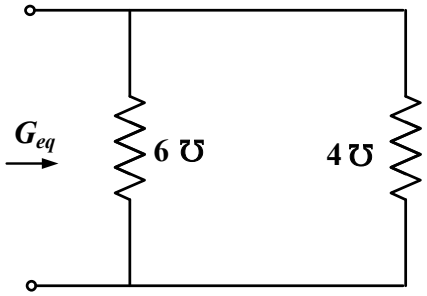
CONTOH 1



$$G_s = \frac{5 \cdot 20}{5 + 20}$$

$$= \frac{100}{25} = 4\ \text{Ʊ}$$



$$G_{eq} = 6 + 4$$

$$= 10\ \text{Ʊ}$$

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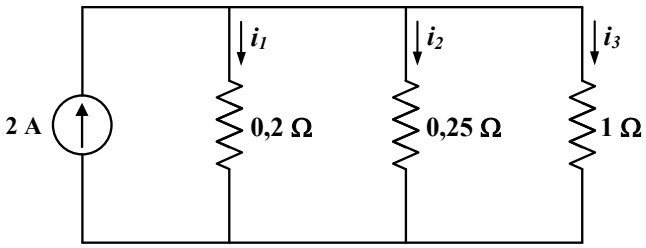
CONTOH 2

Hitunglah arus i_1, i_2 dan i_3 dari rangkaian dengan konduktansi



Solusi

Menghitung konduktasi

$$G_1 = \frac{1}{R_1} \qquad G_2 = \frac{1}{R_2} \qquad G_3 = \frac{1}{R_3}$$

$$G_1 = \frac{1}{0{,}2} = 5\ \text{Ʊ} \qquad G_2 = \frac{1}{0{,}25} = 4\ \text{Ʊ} \qquad G_3 = \frac{1}{1} = 1\ \text{Ʊ}$$

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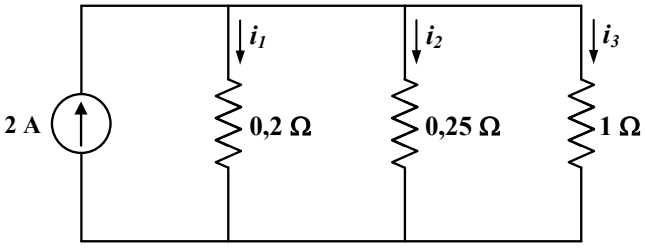
CONTOH 2

Menghitung arus i_1, i_2 dan i_3

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$



$$i_1 = \frac{G_1 i}{G_1 + G_2 + G_3} = \frac{5 \cdot 2}{5 + 4 + 1} = \frac{10}{10} = 1 \text{ A}$$

$$i_2 = \frac{G_2 i}{G_1 + G_2 + G_3} = \frac{4 \cdot 2}{5 + 4 + 1} = \frac{8}{10} = 0{,}8 \text{ A}$$

$$i_3 = \frac{G_3 i}{G_1 + G_2 + G_3} = \frac{1 \cdot 2}{5 + 4 + 1} = \frac{2}{10} = 0{,}2 \text{ A}$$

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NODAL ANALYSIS

- ✓ Analisis nodal *(nodal analysis)* dikenal juga dengan nama metode node-tegangan (*node-voltage method).*
- ✓ Analisis nodal berbasis Hukum Kirchhoff Arus (*Kirchhoff Current Law*).
- ✓ Dalam penyelesaian menggunakan analisis nodal, langkah penting yang harus dilakukan adalah:
  - Pilih satu node sebagai referensi (n ke 0).
  - Pada node lainnya (bukan node referensi), tentukan tegangan v_1, v_2 ……. v_{n-1} terhadap node referensi.
  - Terapkan KCL untuk tiap node (kecuali node referensi) dan gunakan hukum ohm pada setiap arus cabang.

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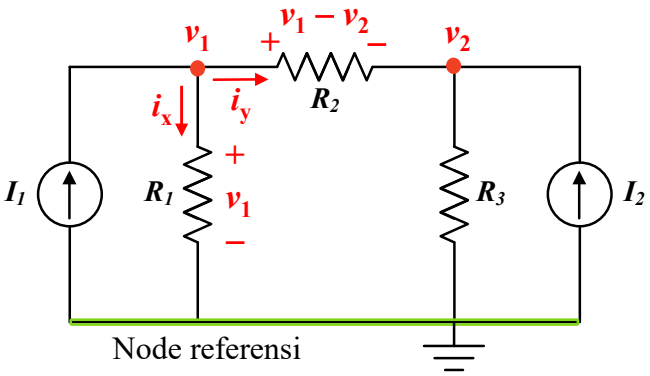
NODAL ANALYSIS → Resistansi R pada Node 1



KCL pada node 1

$I_1 = i_x + i_y$

dimana:

$i_x = \frac{v_1}{R_1}$ dan $i_y = \frac{(v_1 - v_2)}{R_2}$

sehingga:

$I_1 = i_x + i_y$

$$\boldsymbol{I_1 = \frac{v_1}{R_1} + \frac{(v_1 - v_2)}{R_2}}$$

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NODAL ANALYSIS → Konduktansi G pada Node 1

$G_1 = \frac{1}{R_1}$ → $i_x = \frac{v_1}{R_1} = G_1 v_1$

$G_2 = \frac{1}{R_2}$ → $i_y = \frac{(v_1 - v_2)}{R_2} = G_2(v_1 - v_2)$

Dengan demikian

$I_1 = i_x + i_y$

$$\boldsymbol{I_1 = G_1 v_1 + G_2(v_1 - v_2)}$$

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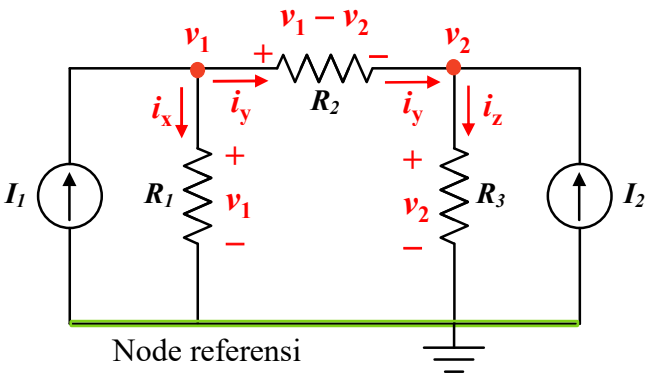
NODAL ANALYSIS → Resistansi R pada Node 2



KCL pada node 2

$i_y + I_2 = i_z$ ⇒ $I_2 = i_z - i_y$

dimana:

$i_z = \frac{v_2}{R_3}$ dan $i_y = \frac{(v_1 - v_2)}{R_2}$

sehingga:

$I_2 = i_z - i_y$

$$\boldsymbol{I_2 = \frac{v_2}{R_3} - \frac{(v_1 - v_2)}{R_2}}$$

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NODAL ANALYSIS → Konduktansi G pada Node 2

$G_3 = \frac{1}{R_3}$ → $i_z = \frac{v_2}{R_3}$ → $= G_3 v_2$

$G_2 = \frac{1}{R_2}$ → $i_y = \frac{(v_1 - v_2)}{R_2}$ → $= G_2(v_1 - v_2)$

Dengan demikian

$I_2 = i_z - i_y$

$$\boldsymbol{I_2 = G_3 v_2 - G_2(v_1 - v_2)}$$

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NODAL ANALYSIS → Resistansi R

Node 1

$$I_1 = \frac{v_1}{R_1} + \frac{(v_1 - v_2)}{R_2}$$

$$I_1 = \frac{v_1}{R_1} + \frac{v_1}{R_2} - \frac{v_2}{R_2}$$

$$I_1 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{v_2}{R_2}$$

Node 2

$$I_2 = \frac{v_2}{R_3} - \frac{(v_1 - v_2)}{R_2}$$

$$I_2 = \frac{v_2}{R_3} - \frac{v_1}{R_2} + \frac{v_2}{R_2}$$

$$I_2 = -\frac{v_1}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2$$

Nilai v_1 dan v_2 dapat dihitung dengan metode:

- Eliminasi
- Substitusi
- Matriks/Cramer

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NODAL ANALYSIS → Resistansi R

$$I_1 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{v_2}{R_2}$$

$$I_2 = -\frac{v_1}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix} I_1 & -\frac{1}{R_2} \\ I_2 & \frac{1}{R_2} + \frac{1}{R_3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{vmatrix}}$$

$$v_2 = \frac{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & I_1 \\ -\frac{1}{R_2} & I_2 \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{vmatrix}}$$

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NODAL ANALYSIS → Konduktansi G

Node 1

$$I_1 = G_1 v_1 + G_2(v_1 - v_2)$$

$$I_1 = G_1 v_1 + G_2 v_1 - G_2 v_2$$

$$I_1 = (G_1 + G_2)\, v_1 - G_2 v_2$$

Node 2

$$I_2 = G_3 v_2 + G_2(v_2 - v_1)$$

$$I_2 = G_3 v_2 + G_2 v_2 - G_2 v_1$$

$$I_2 = -G_2 v_1 + (G_2 + G_3)\, v_2$$

$$I_1 = (G_1 + G_2)\, v_1 - G_2 v_2$$

$$I_2 = -G_2 v_1 + (G_2 + G_3)\, v_2$$

Nilai v_1 dan v_2 dapat dihitung dengan metode:

- ✓ Eliminasi
- ✓ Substitusi
- ✓ Matriks/Cramer

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NODAL ANALYSIS → Konduktansi G

$$I_1 = (G_1 + G_2)\, v_1 - G_2 v_2$$

$$I_2 = -G_2 v_1 + (G_2 + G_3)\, v_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix} I_1 & -G_2 \\ I_2 & G_2 + G_3 \end{vmatrix}}{\begin{vmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{vmatrix}}$$

$$v_2 = \frac{\begin{vmatrix} G_1 + G_2 & I_1 \\ -G_2 & I_2 \end{vmatrix}}{\begin{vmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{vmatrix}}$$

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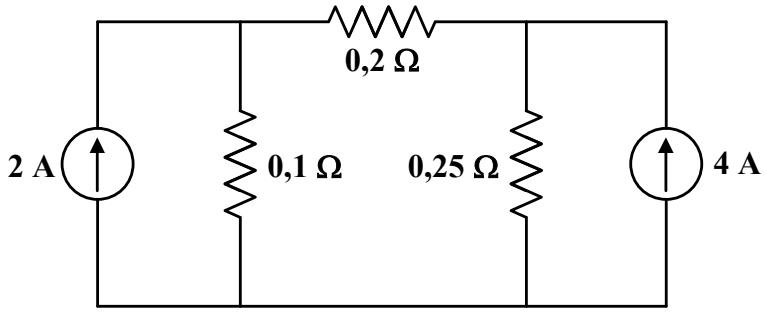
CONTOH 3

Dari rangkaian pada gambar, hitunglah tegangan pada resistor 0,1 Ω dan 0,25 Ω



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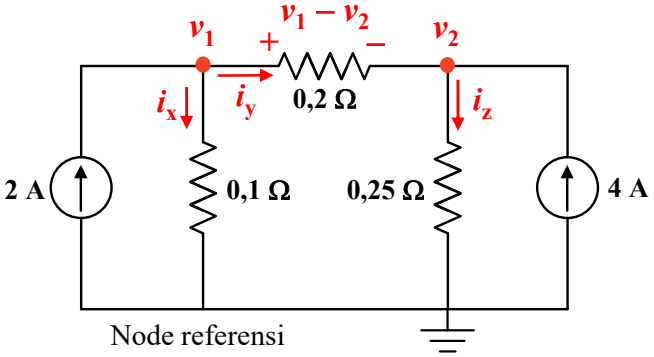
SOLUSI CONTOH 3 → CARA 1



KCL pada node 1

$$I_1 = i_x + i_y$$

$$i_x = \frac{v_1}{0{,}1} = 10v_1 \qquad i_y = \frac{(v_1 - v_2)}{0{,}2} = 5(v_1 - v_2)$$

$$2 = 10v_1 + 5(v_1 - v_2)$$

KCL pada node 2

$$i_y + I_2 = i_z$$

$$I_2 = i_z - i_y$$

$$i_z = \frac{v_2}{0{,}25}$$

$$i_z = 4v_2$$

$$I_2 = i_z - i_y$$

$$4 = 4v_2 - 5(v_1 - v_2)$$

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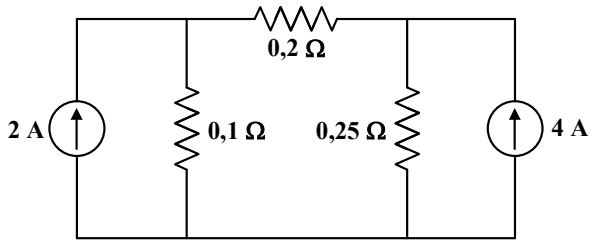
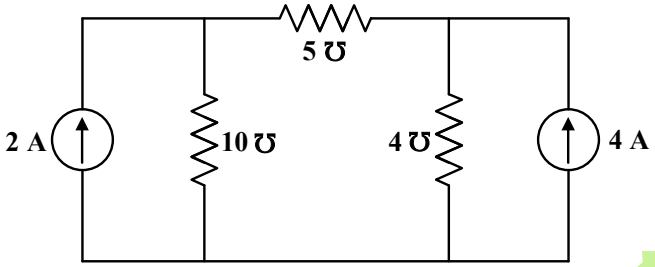
SOLUSI CONTOH 3 → CARA 2



$$G_1 = \frac{1}{0{,}1} = 10 \text{ ℧}$$

$$G_2 = \frac{1}{0{,}2} = 5 \text{ ℧}$$

$$G_3 = \frac{1}{0{,}25} = 4 \text{ ℧}$$



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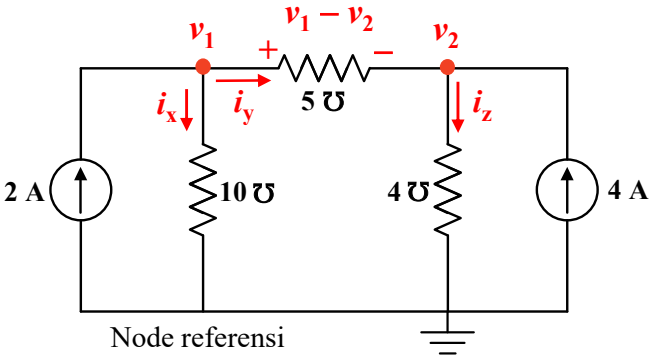
SOLUSI CONTOH 3 → CARA 2



KCL pada node 1

$$I_1 = i_x + i_y$$

$$i_x = 10v_1 \qquad i_y = 5(v_1 - v_2)$$

$$2 = 10v_1 + 5(v_1 - v_2)$$

KCL pada node 2

$$I_2 = i_z - i_y \Rightarrow i_z = 4v_2 \Rightarrow 4 = 4v_2 - 5(v_1 - v_2)$$

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SOLUSI CONTOH 3

Node 1

$$2 = 10v_1 + 5(v_1 - v_2)$$

$$2 = 10v_1 + 5v_1 - 5v_2$$

$$2 = 15v_1 - 5v_2$$

Node 2

$$4 = 4v_2 - 5(v_1 - v_2)$$

$$4 = 4v_2 - 5v_1 + 5v_2$$

$$4 = -5v_1 + 9v_2$$

$$2 = 15v_1 - 5v_2$$

$$4 = -5v_1 + 9v_2$$

Nilai v_1 dan v_2 dapat dihitung dengan metode:

- ✓ Eliminasi
- ✓ Substitusi
- ✓ Matriks/Cramer

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SOLUSI CONTOH 3 → METODE ELIMINASI

$$\begin{array}{l} 2 = 15v_1 - 5v_2 \\ 4 = -5v_1 + 9v_2 \end{array} \quad \left| \begin{array}{l} \times 1 \\ \times 3 \end{array} \right. \quad \begin{array}{l} 2 = 15v_1 - 5v_2 \\ 12 = -15v_1 + 27v_2 \\ \hline 14 = 0 + 22v_2 \end{array} +$$

$$v_2 = \frac{14}{22} = 0{,}636 \text{ V}$$

$$4 = -5v_1 + 9v_2$$

$$4 = -5v_1 + 9(0{,}636)$$

$$5v_1 = 5{,}727 - 4$$

$$v_1 = \frac{1{,}727}{5} = 0{,}345 \text{ V}$$

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SOLUSI CONTOH 3 → METODE CRAMER

$$2 = 15v_1 - 5v_2$$
$$4 = -5v_1 + 9v_2$$

$$\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}15 & -5\\-5 & 9\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix}2 & -5\\4 & 9\end{vmatrix}}{\begin{vmatrix}15 & -5\\-5 & 9\end{vmatrix}} = \frac{18-(-20)}{135-25} = \frac{38}{110} = 0{,}345 \text{ V}$$

$$v_2 = \frac{\begin{vmatrix}15 & 2\\-5 & 4\end{vmatrix}}{\begin{vmatrix}15 & -5\\-5 & 9\end{vmatrix}} = \frac{60-(-10)}{110} = \frac{70}{110} = 0{,}636 \text{ V}$$

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