



# ALJABAR LINIER

## PENYELESAIAN KUIS 1

Wike Handini

### SOAL 1 → Sub CPMK 1

Hitunglah nilai  $x_1$ ,  $x_2$  dan  $x_3$  dari SPL berikut:

$$\begin{aligned} -2x_1 + 2x_2 - 2x_3 &= -4 \\ 3x_1 - 4x_2 + x_3 &= -2 \\ 2x_1 - x_2 + 2x_3 &= 6 \end{aligned}$$

dengan menggunakan:

- Invers Matriks
- Kaidah Cramer

Penyelesaian a

$$-2x_1 + 2x_2 - 2x_3 = -4$$

$$3x_1 - 4x_2 + x_3 = -2$$

$$2x_1 - x_2 + 2x_3 = 6$$



$$\begin{bmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$



$$A = \begin{bmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

## SOAL 1 → INVERS MATRIKS

Minor (A)

$$A = \begin{bmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

$$m_{31} = \begin{vmatrix} 2 & -2 \\ -4 & 1 \end{vmatrix} = 2 - 8 = -6$$

$$m_{11} = \begin{vmatrix} -4 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= -8 + 1 = -7$$

$$m_{21} = \begin{vmatrix} 2 & -2 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 2 = 2$$

$$m_{32} = \begin{vmatrix} -2 & -2 \\ 3 & 1 \end{vmatrix} = -2 + 6 = 4$$

$$m_{12} = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= 6 - 2 = 4$$

$$m_{22} = \begin{vmatrix} -2 & -2 \\ 2 & 2 \end{vmatrix}$$

$$= -4 + 4 = 0$$

$$m_{33} = \begin{vmatrix} -2 & 2 \\ 3 & -4 \end{vmatrix} = 8 - 6 = 2$$

$$m_{13} = \begin{vmatrix} 3 & -4 \\ 2 & -1 \end{vmatrix}$$

$$= -3 + 8 = 5$$

$$m_{23} = \begin{vmatrix} -2 & 2 \\ 2 & -1 \end{vmatrix}$$

$$= 2 - 4 = -2$$

$$\text{Minor (A)} = \begin{bmatrix} -7 & 4 & 5 \\ 2 & 0 & -2 \\ -6 & 4 & 2 \end{bmatrix}$$

3

## SOAL 1 → INVERS MATRIKS

$$\text{Minor (A)} = \begin{bmatrix} -7 & 4 & 5 \\ 2 & 0 & -2 \\ -6 & 4 & 2 \end{bmatrix}$$



$$\text{Kofaktor (A)} = \begin{bmatrix} -7 & -4 & 5 \\ -2 & 0 & 2 \\ -6 & -4 & 2 \end{bmatrix}$$



$$|A| = (-2 \cdot 3) + (0 \cdot -4) + (2 \cdot 1)$$

$$= -6 + 2 = -4$$

$$A = \begin{bmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$



$$\text{Adjoin (A)} = \begin{bmatrix} -7 & -4 & 5 \\ -2 & 0 & 2 \\ -6 & -4 & 2 \end{bmatrix}^T = \begin{bmatrix} -7 & -2 & -6 \\ -4 & 0 & -4 \\ 5 & 2 & 2 \end{bmatrix}$$

4

## SOAL 1 → INVERS MATRIKS

Invers (A)

$$A^{-1} = \frac{1}{\det(A)} \text{Adjoin (A)}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} -7 & -2 & -6 \\ -4 & 0 & -4 \\ 5 & 2 & 2 \end{bmatrix} = -0,25 \begin{bmatrix} -7 & -2 & -6 \\ -4 & 0 & -4 \\ 5 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -0,25(-7) & -0,25(-2) & -0,25(-6) \\ -0,25(-4) & -0,25 \cdot 0 & -0,25(-4) \\ -0,25 \cdot 5 & -0,25 \cdot 2 & -0,25 \cdot 2 \end{bmatrix} = \begin{bmatrix} 1,75 & 0,5 & 1,5 \\ 1 & 0 & 1 \\ -1,25 & -0,5 & -0,5 \end{bmatrix}$$

5

## SOAL 1 → INVERS MATRIKS

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$



$$x_1 = 1$$

$$x_2 = 2$$

$$x_3 = 3$$

$$= \begin{bmatrix} 1,75 & 0,5 & 1,5 \\ 1 & 0 & 1 \\ -1,25 & -0,5 & -0,5 \end{bmatrix} \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1,75(-4) + 0,5(-2) + 1,5 \cdot 6 \\ 1(-4) + 0(-2) + 1 \cdot 6 \\ -1,25(-4) + (-0,5)(-2) + (-0,5)6 \end{bmatrix} = \begin{bmatrix} -7 - 1 + 9 \\ -4 + 0 + 6 \\ 5 + 1 - 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

6

## SOAL 1 → Kaidah Cramer

$$\begin{aligned} -2x_1 + 2x_2 - 2x_3 &= -4 \\ 3x_1 - 4x_2 + x_3 &= -2 \\ 2x_1 - x_2 + 2x_3 &= 6 \end{aligned} \Rightarrow \begin{bmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} -4 & 2 & -2 \\ -2 & -4 & 1 \\ 6 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix}}$$

$$x_2 = \frac{\begin{vmatrix} -2 & -4 & -2 \\ 3 & -2 & 1 \\ 2 & 6 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix}}$$

$$x_3 = \frac{\begin{vmatrix} -2 & 2 & -4 \\ 3 & -4 & -2 \\ 2 & -1 & 6 \end{vmatrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix}}$$

7

## SOAL 1 → Kaidah Cramer

$$\begin{aligned} x_1 &= \frac{\begin{vmatrix} -4 & 2 & -2 \\ -2 & -4 & 1 \\ 6 & -1 & 2 \end{vmatrix} \begin{matrix} + \\ - \\ + \end{matrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix} \begin{matrix} + \\ - \\ + \end{matrix}} = \frac{-2 \begin{vmatrix} -2 & -4 \\ 6 & -1 \end{vmatrix} + (-1) \begin{vmatrix} -4 & 2 \\ 6 & -1 \end{vmatrix} + 2 \begin{vmatrix} -4 & 2 \\ -2 & -4 \end{vmatrix}}{-2 \begin{vmatrix} 3 & -4 \\ 2 & -1 \end{vmatrix} + (-1) \begin{vmatrix} -2 & 2 \\ 2 & -1 \end{vmatrix} + 2 \begin{vmatrix} -2 & 2 \\ 3 & -4 \end{vmatrix}} \\ &= \frac{-2(2 + 24) - 1(4 - 12) + 2(16 + 4)}{-2(-3 + 8) - 1(2 - 4) + 2(8 - 6)} \\ &= \frac{-2(26) - 1(-8) + 2(20)}{-2(5) - 1(-2) + 2(2)} \\ &= \frac{-52 + 8 + 40}{-10 + 2 + 4} = \frac{-4}{-4} = 1 \end{aligned}$$

8

### SOAL 1 → Kaidah Cramer

$$\begin{aligned}
 x_2 &= \frac{\begin{vmatrix} -2 & -4 & -2 \\ 3 & -2 & 1 \\ 2 & 6 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix}} = \frac{-2 \begin{vmatrix} 3 & -2 \\ 2 & 6 \end{vmatrix} - 1 \begin{vmatrix} -2 & -4 \\ 2 & 6 \end{vmatrix} + 2 \begin{vmatrix} -2 & -4 \\ 3 & -2 \end{vmatrix}}{-4} \\
 &= \frac{-2(18 + 4) - 1(-12 + 8) + 2(4 + 12)}{-4} \\
 &= \frac{-2(22) - 1(-4) + 2(16)}{-4} \\
 &= \frac{-44 + 4 + 32}{-4} = \frac{-8}{-4} = 2
 \end{aligned}$$

9

### SOAL 1 → Kaidah Cramer

$$\begin{aligned}
 x_3 &= \frac{\begin{vmatrix} -2 & 2 & -4 \\ 3 & -4 & -2 \\ 2 & -1 & 6 \end{vmatrix}}{\begin{vmatrix} -2 & 2 & -2 \\ 3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix}} = \frac{-2 \begin{vmatrix} 3 & -2 \\ 2 & 6 \end{vmatrix} - 4 \begin{vmatrix} -2 & -4 \\ 2 & 6 \end{vmatrix} - (-1) \begin{vmatrix} -2 & -4 \\ 3 & -2 \end{vmatrix}}{-4} \\
 &= \frac{-2(18 + 4) - 4(-12 + 8) + 1(4 + 12)}{-4} \\
 &= \frac{-2(22) - 4(-4) + 1(16)}{-4} \\
 &= \frac{-44 + 16 + 16}{-4} = \frac{-12}{-4} = 3
 \end{aligned}$$

10

## SOAL 2 → Sub CPMK 2

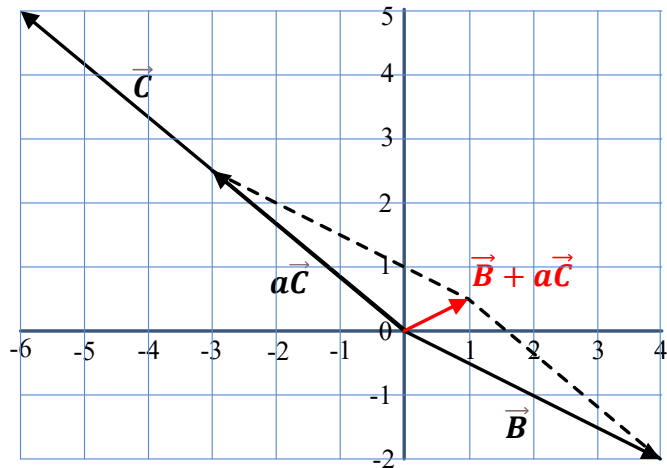
Jika diketahui nilai skalar  $a = 0,5$  dan vektor  $B = (4, -2)$  serta vektor  $C = (-6, 5)$ , cari dan gambarkan nilai vektor dari:

a.  $\vec{B} + a\vec{C}$

b.  $\vec{B} - a\vec{C}$

Solusi a

$$\begin{aligned}\vec{B} + a\vec{C} &= (4, -2) + 0,5 \cdot (-6, 5) \\ &= (4, -2) + (-3, 2,5) \\ &= (1, 0,5)\end{aligned}$$



## SOAL 2 → Sub CPMK 2

Solusi b

$$\begin{aligned}\vec{B} - a\vec{C} &= \vec{B} + (-a\vec{C}) \\ -a\vec{C} &= -(-3, 2,5) \\ &= (3, -2,5) \\ \vec{B} + (-a\vec{C}) &= (4, -2) + (3, -2,5) \\ &= (7, -4,5)\end{aligned}$$

