



ALJABAR LINIER

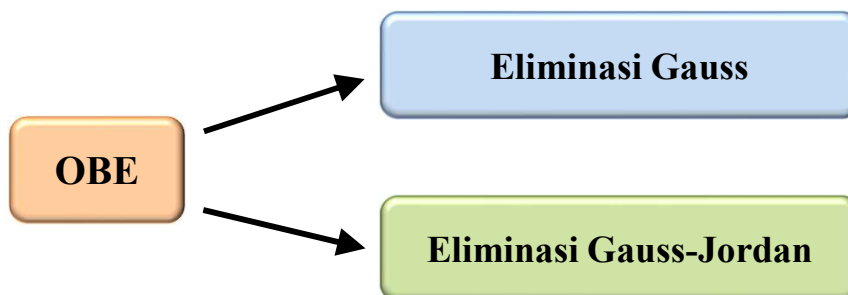
Sistem Persamaan Linier (Bagian 2)

Wike Handini

MATRIKS → OPERASI BARIS ELEMENTER (OBE)

- ✓ Operasi mengalikan suatu baris dengan bilangan bukan nol.
- ✓ Operasi menambahkan/mengurangkan kelipatan suatu baris pada baris lain.
- ✓ Operasi menukarkan sembarang dua buah baris.

OBE



PENERAPAN MATRIKS DALAM SISTEM PERSAMAAN LINIER

$$\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13000 \\ 9000 \end{bmatrix}$$

$$\mathbf{A} \mathbf{X} = \mathbf{B}$$

$$\text{Matriks A} = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$$

Menentukan matriks yang diperbesar

Penyelesaian dengan OBE

$$\left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 1 & 3 & 9000 \end{array} \right]$$

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PENYELESAIAN → METODE ELIMINASI GAUSS

$$\left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 1 & 3 & 9000 \end{array} \right]$$

Baris dua dikurangi 1/3 kali baris satu

Nilai
0

$$\left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 1 - \frac{1}{3}3 & 3 - \frac{1}{3}2 & 9000 - \frac{1}{3}13000 \end{array} \right] = \left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

maka sistem persamaan liniernya menjadi:

$$\begin{bmatrix} 3 & 2 \\ 0 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13000 \\ \frac{14000}{3} \end{bmatrix}$$

Matriks segitiga atas

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PENYELESAIAN → METODE ELIMINASI GAUSS

$$\begin{bmatrix} 3 & 2 \\ 0 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13000 \\ \frac{14000}{3} \end{bmatrix}$$

$$3x + 2y = 13000$$

$$\frac{7}{3}y = \frac{14000}{3}$$

Substitusi

Metode substitusi balik

$$\frac{7}{3}y = \frac{14000}{3}$$

$$3 \times \frac{7}{3}y = \frac{14000}{3} \times 3$$

$$7y = 14000$$

$$y = \frac{14000}{7} = 2000$$

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PENYELESAIAN → METODE ELIMINASI GAUSS

$$3x + 2y = 13000$$

$$3x + 2(2000) = 13000$$

$$3x + 4000 = 13000$$

$$3x = 13000 - 4000$$

$$3x = 9000$$

$$x = \frac{9000}{3} = 3000$$

Dengan demikian
didapatkan hasilnya
adalah:

$$x = 3000$$

$$y = 2000$$

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PENYELESAIAN → METODE ELIMINASI GAUSS-JORDAN

$$\left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 1 & 3 & 9000 \end{array} \right]$$



$$\left[\begin{array}{cc|c} 1 & 0 & M \\ 0 & 1 & N \end{array} \right]$$



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} M \\ N \end{bmatrix}$$



$$x = M$$

$$y = N$$



$$x + 0y = M$$

$$0x + y = N$$

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PENYELESAIAN → METODE ELIMINASI GAUSS-JORDAN

Nilai 0

$$\left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 1 & 3 & 9000 \end{array} \right] = \left[\begin{array}{cc|c} 3 & 2 & 13000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$



Baris satu dikurangi 6/7 kali baris dua



$$\left[\begin{array}{cc|c} 3 - 0 & 2 - \left(\frac{6}{7} \times \frac{7}{3}\right) & 13000 - \left(\frac{6}{7} \times \frac{14000}{3}\right) \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right] = \left[\begin{array}{cc|c} 3 & 2 - 2 & 13000 - 4000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

$$= \left[\begin{array}{cc|c} 3 & 0 & 9000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

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PENYELESAIAN → METODE ELIMINASI GAUSS-JORDAN

Nilai 1

$$\left[\begin{array}{cc|c} 3 & 0 & 9000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

Baris satu dikali 1/3

$$\left[\begin{array}{cc|c} 3 \times \frac{1}{3} & 0 & 9000 \times \frac{1}{3} \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right] = \left[\begin{array}{cc|c} 1 & 0 & 3000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

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PENYELESAIAN → METODE ELIMINASI GAUSS-JORDAN

$$\left[\begin{array}{cc|c} 1 & 0 & 3000 \\ 0 & \frac{7}{3} & \frac{14000}{3} \end{array} \right]$$

Baris dua dikali 3/7

$$\left[\begin{array}{cc|c} 1 & 0 & 3000 \\ 0 & \frac{7}{3} \times \frac{3}{7} & \frac{14000}{3} \times \frac{3}{7} \end{array} \right]$$

Nilai 1

$$\left[\begin{array}{cc|c} 1 & 0 & 3000 \\ 0 & 1 & 2000 \end{array} \right]$$

$$x = 3000$$

$$y = 2000$$

$$x + 0y = 3000$$

$$0x + y = 2000$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3000 \\ 2000 \end{bmatrix}$$

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DETERMINAN MATRIKS

- ✓ Determinan matriks A bisa ditulis dengan tanda **det (A)**, **det A**, atau **|A|**
- ✓ Determinan adalah suatu nilai matriks yang berbentuk persegi
- ✓ Determinan matriks hanya dimiliki oleh sebuah matriks yang jumlah kolom dan jumlah barisnya sama

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DETERMINAN MATRIKS ORDE 2 x 2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \Rightarrow \quad \det A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

CONTOH

$$A = \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix} \quad \Rightarrow \quad |A| = 5 \times (-2) - 3 \times 4 = -10 - 12 = -22$$

$$A = \begin{bmatrix} 6 & 3 \\ -7 & \frac{1}{2} \end{bmatrix} \quad \Rightarrow \quad |A| = 6 \times \left(\frac{1}{2}\right) - (-7) \times 3 = 3 + 21 = 24$$

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DETERMINAN MATRIKS ORDE 3 x 3 → METODE SARRUS

Salin kolom pertama dan kedua matriks A ke sebelah kanan tanda determinan



$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$



$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix}$$

$$= (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$

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CONTOH

Hitunglah determinan dari matriks $A = \begin{bmatrix} 4 & -2 & 6 \\ 8 & 3 & 1 \\ 2 & 7 & -5 \end{bmatrix}$

Solusi

$$|A| = \begin{vmatrix} 4 & -2 & 6 \\ 8 & 3 & 1 \\ 2 & 7 & -5 \end{vmatrix} = \begin{vmatrix} 4 & -2 & 6 & 4 & -2 \\ 8 & 3 & 1 & 8 & 3 \\ 2 & 7 & -5 & 2 & 7 \end{vmatrix}$$

$$= [(4 \times 3 \times -5) + (-2 \times 1 \times 2) + (6 \times 8 \times 7)] - [(2 \times 3 \times 6) + (7 \times 1 \times 4) + (-5 \times 8 \times -2)]$$

$$= [-60 + (-4) + 336] - [36 + 28 + 80]$$

$$= 272 - 144 = 128$$

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